PHYSICAL SOCIETY

MEMOIRS

OF THE

COLLEGE OF SCIENCE

KYOTO IMPERIAL UNIVERSITY

SERIES A

VOL. XIX. NO. 3

MAY 1936

KYOTO

A Study of the Effect of a Local Wind upon the Sea-Surface and on the Development of the Internal Boundary Wave

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(Received December 31, 1935)

Abstract

In the present paper,[1] from a simple theoretical calculation, the writer first investigates the elevation or depression of the sea-surface and the variation of the total flow of the current generated by a *travelling local* gale of large scale (such as a cyclone), and secondly the behaviour of the generation of the internal long wave of great height.

Introduction

The motion of sea-water produced by a wind action has been investigated by Prof. T. Nomitsu,[2] Dr. K. Hidaka[3] and recently by Mr. G. Nishimura.[4] In this paper the writer will study the elevation or depression of the sea-surface generated by a *local* gale of large scale and of the development of the internal boundary wave (the elevation or depression of the internal surface discontinuous regarding the density).

In Chap. I. we give the fundamental equations and the conditions which are adequate for the present problem. In Chap. II. their solutions will be given, and moreover to give a concrete explanation of the solutions, we will calculate the following special cases: (1). The region, in which the wind-velocity is uniform, *travels* with a velocity V on the *unbounded* ocean of uniform depth. (2). The ocean is bounded by a *coast* and (a) a wind region travels towards the coast from the sea, (b) it travels towards the sea from the coast.

Specially, case (a) of (2) may be one of the causes of the abnormally high water (e. g. at Muroto Sept. 21, 1934) produced by a strong typhoon.

In Chap. III. we will give details of the mechanisms of the development of the internal boundary waves by taking a simple example as wind region.

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1. An outline of this paper was read at the Annual Meeting of Physico-Mathematical Society of Japan in April 1935. 2. These Memoirs, A, **17**, 249 (1934). 3. Geophy. Mag. Cent. Meteo. Obs. Tokyo. **7**, No. 3-4, 1933. 4. 地震研究所彙報第二號(昭和十年五月).

But, here, it must be noticed that the present problems are all treated in two dimensions (vertical and horizontal directions) without consideration of the effect of the earth's rotation, and the assumed boundary conditions are that no friction exists at the bottom or at the boundary surface.

I. Fundamental Equations and Conditions

Take the x-axis on the undisturbed sea-surface and the z-axis vertically upwards. Let us denote

g: acceleration of the gravity;
ρ: density of the sea water (assumed to be unity);
h: depth of ocean;
t: time;
u: velocity of the current in x-direction;
μ: coefficient of the eddy viscosity (assumed to be constant everywhere);
ζ: elevation or depression from the undisturbed surface;
T: wind traction (assumed to exist in a region of breath $2l$ only).

If the small terms may be neglected the Navier equation of motion and the equation of continuity may be written as follows.

$$\frac{\partial u}{\partial t}=\mu\frac{\partial^2 u}{\partial z^2}-g\frac{\partial \zeta}{\partial x} \qquad \cdots\cdots(1),$$

$$\frac{\partial}{\partial x}\int_{-h}^{0} u dz=-\frac{\partial \zeta}{\partial t} \qquad \cdots\cdots(2).$$

The surface and bottom conditions are assumed to be

$$\left.\begin{aligned} \mu\frac{\partial u}{\partial z}&=T \qquad \text{at } z=0\\ \mu\frac{\partial u}{\partial z}&=0 \qquad \text{at } z=-h \end{aligned}\right\} \qquad \cdots\cdots(3),$$

where $T=f(x, t)$ within the wind region $2l$, $T=0$ outside the region.

Now integrate (1) with respect to z from $-h$ to 0, and take into account eq. (3), then we have

$$\frac{\partial S}{\partial t}=T-gh\frac{\partial \zeta}{\partial x} \qquad \cdots\cdots(1'),$$

where $S=\int_{-h}^{0} u dz$ represents the total flow perpendicular to the vertical section of unit width due to the drift- and slope-current.

By using the total flow S, the equation of continuity (2) may be written as

$$\frac{\partial S}{\partial x}=-\frac{\partial \zeta}{\partial t} \qquad \cdots\cdots(2').$$

Eliminate ζ from (1′) and (2′), then we have

$$\frac{\partial^2 S}{\partial t^2}=gh\frac{\partial^2 S}{\partial x^2}+\frac{\partial T}{\partial x} \qquad \cdots\cdots(4).$$

This is a fundamental equation for the present problem. The initial and boundary conditions are as follows.

$$\left.\begin{aligned} &S=0 \text{ and } \frac{\partial S}{\partial t}=T \left(\because \frac{\partial \zeta}{\partial x}=0\right) \text{ when } t=0 \\ &S \text{ is continuous at the ends of the wind region} \end{aligned}\right\} \quad \cdots(5),$$

and when the ocean is bounded by a coast

$$S=0 \quad \text{at the coast } (x=0). \qquad \cdots\cdots(6).$$

Now, if the solution S of eq. (4) satisfying conditions (5) and (6) can be obtained, substitute it into (2′); then ζ may be obtained by integrating it and using the condition that $\zeta=0$ when $t=0$.

Conversely, if S is eliminated from (1′) and (2′), the fundamental equation for ζ may be obtained as

$$\frac{\partial^2 \zeta}{\partial t^2}=\frac{\partial}{\partial x}\left(gh\frac{\partial \zeta}{\partial x}\right)-\frac{\partial T}{\partial x} \qquad \cdots\cdots(4')$$

and the initial and boundary conditions corresponding to (5) and (6) will be

$$\left.\begin{aligned} &\zeta=0 \text{ and } \frac{\partial \zeta}{\partial t}=0 \text{ when } t=0 \\ &gh\left(\frac{\partial \zeta_T}{\partial x}-\frac{\partial \zeta_0}{\partial x}\right)=T \quad \text{at the end of wind region} \end{aligned}\right\} \quad \cdots\cdots(5'),$$

where ζ_T and ζ_0 express the elevations corresponding to the regions where T exists and does not exist respectively. When the ocean is bounded by a coast, the condition on the coast is

$$gh\frac{\partial \zeta}{\partial x}=T \qquad \text{at the coast } (x=0) \qquad \cdots\cdots(6').$$

Now, referring to eq. (1′) it must be here noticed that, as long as we use eq. (1), the solutions, which satisfy the condition such that the current u is nil on the coast and is continuous at the place where T changes abruptly, cannot generally be obtained. But if we use eq. (4) or (4′) instead of (1), without considering current u, we may obtain solutions appropriate mathematically and also physically, except for the current u in the neighbourhood of the coast. Thus the coast condition should be put $S=0$ or $gh\frac{\partial \zeta}{\partial x}=T$, but not $u=0$ or $\frac{\partial \zeta}{\partial x}=0$ as assumed by some authors.

Since eq. (4) and (4′) are obtained from the assumption of no bottom-friction and consequently contain no damping terms, the solution will afford only a quasi-steady state but cannot afford a steady state.

For practical purposes, the solution in the case of a bottom-friction is more desirable. If we assume the bottom condition as $\mu\frac{\partial u}{\partial z}=k\underline{u}$, then corresponding to (1′) we get

$$\frac{\partial S}{\partial t}=T-k\underline{u}-gh\frac{\partial \zeta}{\partial x} \qquad \cdots\cdots\cdots\cdots(1''),$$

and corresponding to (4)

$$\frac{\partial^2 S}{\partial t^2}+k\frac{\partial u}{\partial t}=gh\frac{\partial^2 S}{\partial x^2}+\frac{\partial T}{\partial t} \qquad \cdots\cdots\cdots\cdots(4''),$$

is obtained, where $\underline{u}$ and k means the bottom velocity and the proportional constant respectively. If we assume suitably $\underline{u}$ as a function of S, we may obtain an approximate solution of S. For instance, in the case of an infinitely long canal, as $\underline{u}$ may be put to be proportional to S approximately, (4″) becomes

$$\frac{\partial^2 S}{\partial t^2}+2f\frac{\partial S}{\partial t}=gh\frac{\partial^2 S}{\partial x^2}+\frac{\partial T}{\partial t} \qquad \cdots\cdots\cdots\cdots(4''').$$

The rigorous solution of (4‴) is very troublesome, and we will develop our discussion only with eq. (4) in this paper.

II. Solutions

Case. 1. Unbounded Ocean

Solve eq. (4) using the condition (5), then we have

$$S=\frac{1}{\pi}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty} T(\lambda,\tau)\cos\alpha c(t-\tau)\cos\alpha(\lambda-x)d\lambda \quad \cdots\cdots(7),$$

and from (2′)

$$\zeta=-\frac{1}{c\pi}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty} T(\lambda,\tau)\sin\alpha c(t-\tau)\sin\alpha(\lambda-x)d\lambda \quad \cdots(8),$$

where $c=\sqrt{gh}$. From these it may be seen that the mode of the motion produced propagates with the long wave velocity towards right and left from the origin.

Now, especially, from the consideration of the following wind region for which

$T(x,t)=$const. for $-l<x-Vt<l$
$T=0$ for $x-Vt>l$ and $x-Vt<-l$ } during $0<t<t_1$
and $T=0$ all over the sea-surface after $t>t_1$,

we will give a concrete example of the effect of the bounded wind region travelling with constant velocity.

For $t_1>t>0$, eqs. (7) and (8) give

$$S=\frac{2TV}{\pi(c^2-V^2)}\int_0^\infty\frac{\sin\alpha(x-Vt)\sin\alpha l}{\alpha^2}d\alpha+\frac{T}{\pi(c+V)}\int_0^\infty\frac{\sin\alpha(x+ct)\sin\alpha l}{\alpha^2}d\alpha$$
$$-\frac{T}{\pi(c-V)}\int_0^\infty\frac{\sin\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha \qquad \ldots\ldots\ldots\ldots\ldots\ldots(9),$$

$$\zeta=\frac{2T}{\pi(c^2-V^2)}\int_0^\infty\frac{\sin\alpha(x-Vt)\sin\alpha l}{\alpha^2}d\alpha-\frac{T}{\pi c(c+V)}\int_0^\infty\frac{\sin\alpha(x+ct)\sin\alpha l}{\alpha^2}d\alpha$$
$$-\frac{T}{\pi c(c-V)}\int_0^\infty\frac{\sin\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha \qquad \ldots\ldots\ldots\ldots\ldots\ldots(9'),$$

and for $t>t_1$

$$S=\frac{T}{\pi(c-V)}\int_0^\infty\frac{\sin\alpha(x-Vt_1-c\overline{t-t_1})\sin\alpha l}{\alpha^2}d\alpha$$
$$-\frac{T}{\pi(c+V)}\int_0^\infty\frac{\sin\alpha(x-Vt_1+c\overline{t-t_1})\sin\alpha l}{\alpha^2}d\alpha$$
$$+\frac{T}{\pi(c-V)}\int_0^\infty\frac{\sin\alpha(x+ct)\sin\alpha l}{\alpha^2}d\alpha-\frac{T}{\pi(c-V)}\int_0^\infty\frac{\sin\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10),$$

$$\zeta=\frac{T}{\pi c(c-V)}\int_0^\infty\frac{\sin\alpha(x-Vt_1-c\overline{t-t_1})\sin\alpha l}{\alpha^2}d\alpha$$
$$+\frac{T}{\pi c(c+V)}\int_0^\infty\frac{\sin\alpha(x-Vt_1+c\overline{t-t_1})\sin\alpha l}{\alpha^2}d\alpha$$
$$-\frac{T}{\pi c(c-V)}\int_0^\infty\frac{\sin\alpha(x+ct)\sin\alpha l}{\alpha^2}d\alpha-\frac{T}{\pi c(c-V)}\int_0^\infty\frac{\sin\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10').$$

Now, carrying out the integration of eqs. (9) and (9′) from a well-known formula $\left(\int_0^\infty\frac{\sin ax\sin bx}{x^2}dx=\frac{\pi}{2}a$ for $0<a<b$, and $=\frac{\pi}{2}b$ for $0<b<a\right)$, we have the following results for S and ζ:

1st term of S and $\zeta=\frac{VT}{c^2-V^2}(x-Vt)$ and $\frac{T}{c^2-V^2}(x-Vt)$
for $-l<x-Vt<l$,

" " $=\frac{\pm VT}{c^2-V^2}l$ and $\frac{\pm T}{c^2-V^2}l$
for $x-Vt>l$, $x-Vt<-l$;

2nd term of these$=\frac{T}{2(c+V)}(x+ct)$ and $\frac{T}{2c(c+V)}(x+ct)$
for $-l<x+ct<l$,

$$\text{,,} \qquad \text{,,} \qquad = \frac{\pm T}{2(c+V)} l \text{ and } \frac{\mp T}{2c(c+V)} l$$

for $x-ct>l$, $x-ct<-l$

$$\text{3rd term of these} = \frac{T}{2(c-V)}(x-ct) \text{ and } \frac{T}{2c(c-V)}(x-ct)$$

for $-l<x-ct<l$,

$$\text{,,} \qquad \text{,,} \qquad = \frac{\mp T}{2(c-V)} l \text{ and } \frac{\mp T}{2c(c-V)} l$$

for $x-ct>l$, $x-ct<-l$,

respectively.

From these formulae we know that the 1st term expresses the forced wave (due to the wind action) to be propagated with a velocity V towards the positive direction of x (travelling direction of the wind

Fig. 1a.

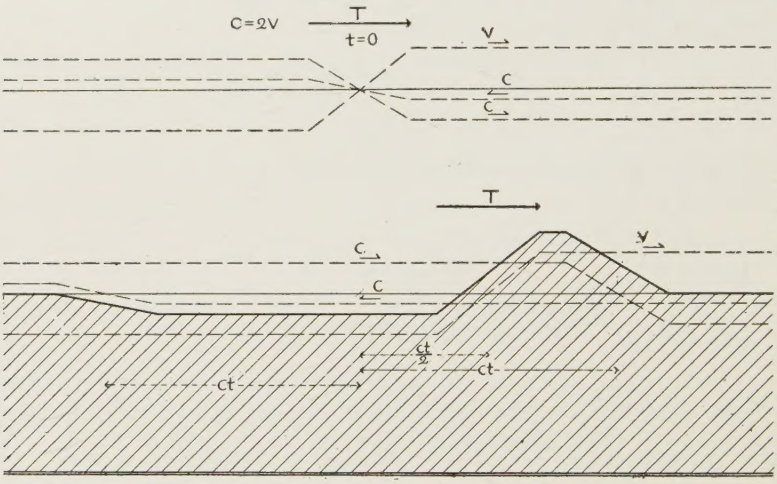
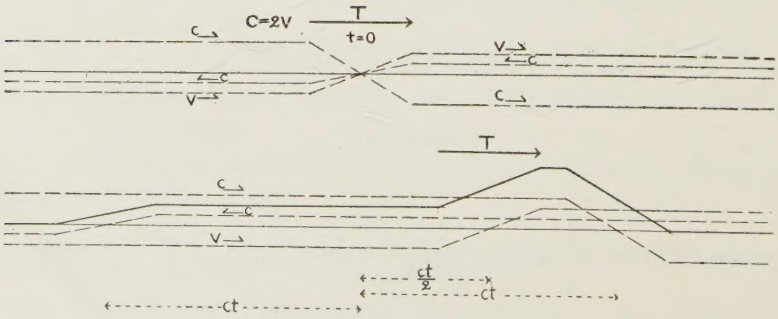


Fig. 1b.



region), and the 2nd and 3rd terms express the increase of the waves of free wave velocity $c=\sqrt{gh}$ towards the positive and the negative direction respectively. The general features of ζ and S are shown in Figs. 1a and 1b.

Of course, in the region of a cyclone the wind directions on both sides of the direction in which its centre is travelling are generally opposite to one another, but if we consider only one side, the case seems to belong to the present one.

After the wind traction has become nil (practically, after the force of a cyclone has become very small) S and ζ are given by (10) and (10′) respectively, but here to avoid redundancy we give no concrete explanation for this.

The formulae (9), (9′) and Figs. 1a and 1b show that the nearer the travelling velocity V approaches to the free wave velocity c the larger ζ and S become with time, and the height of the front part of the wave is greater than that of the rear part. But even if V is equal to c they cannot grow infinite instantly[1], as at a first sight. When V is equal to c, from the theory of differential equation the formulae corresponding to (9), (9′), (10) and (10′) will become as follows.

For $0<t<t_1$,

$$S=\frac{T}{\pi}\int_0^\infty \frac{t\cos\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha+\frac{T}{2\pi c}\int_0^\infty \frac{\sin\alpha(x+ct)\sin\alpha l}{\alpha^2}d\alpha$$
$$-\frac{T}{2\pi c}\int_0^\infty \frac{\sin\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha \qquad \text{.....(9″)},$$

$$\zeta=\frac{T}{\pi c}\int_0^\infty \frac{t\cos\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha+\frac{T}{2\pi c^2}\int_0^\infty \frac{\sin\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha$$
$$-\frac{T}{2\pi c^2}\int_0^\infty \frac{\sin\alpha(x+ct)\sin\alpha l}{\alpha^2}d\alpha \qquad \text{.....(9‴)},$$

and for $t>t_1$,

$$S=\frac{T}{\pi}\int_0^\infty \frac{t_1\cos\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha+\frac{T}{2\pi c}\int_0^\infty \frac{\sin\alpha(x+ct)\sin\alpha l}{\alpha^2}d\alpha$$
$$-\frac{T}{2\pi c}\int_0^\infty \frac{\sin\alpha(x+ct-2ct_1)\sin\alpha l}{\alpha^2}d\alpha \qquad \text{.....(10″)},$$

$$\zeta=\frac{T}{\pi c}\int_0^\infty \frac{t_1\cos\alpha(x-ct)\sin\alpha l}{\alpha^2}d\alpha-\frac{T}{2\pi c^2}\int_0^\infty \frac{\sin\alpha(x+ct)\sin\alpha l}{\alpha^2}d\alpha$$

1. Moreover, when the friction is considered at the bottom, ζ and S cannot become infinitely large even after a very long time and even if V approaches c. This point has been shown recently by Prof. T. Nomitsu from a simple calculation, These Memoirs, A, **18**, 211 (1935).

$$+\frac{T}{2\pi c^2}\int_0^\infty \frac{\sin a(x+ct-2ct_1)\sin al}{a^2}da \qquad \ldots\ldots\ldots\ldots(10''').$$

These formulae (9″), (9‴), (10″) and (10‴) show that ζ and S increase with time from zero, and that, in order to grow very large, T must continue for a fairly long time, namely the modes of the development are similar to the resonance phenomena in Acaustic.[1]

Case 2. Ocean bounded by a Straight Coast

Take the x-axis such that its origin lies on a coast and x is measured from the coast seawards. Solve eq. (4) under the conditions (5) (in this case supposing that the wind blows towards the coast, then (5) becomes $\frac{\partial S}{\partial t}=-T$) and (6), then we have

$$S=\frac{-2}{\pi}\int_0^t d\tau\int_0^\infty da\int_0^\infty T\sin ax\sin a\lambda\cos ac(t-\tau)d\lambda \quad \ldots(11),$$

$$\zeta=\frac{2}{\pi c}\int_0^t d\tau\int_0^\infty da\int_0^\infty T\cos ax\sin a\lambda\sin ac(t-\tau)d\lambda \quad \ldots(12).$$

Now, especially, we shall consider the same wind region as in the preceding case except the wind direction is opposite.

(*a*). *The case of the wind region travelling from the sea towards the coast*

We shall take the case where a wind region of width $2l$ suddenly generates at $x=a$ over the sea and advances towards the coast. Moreover, it seems to be appropriate to take the distance a as equal to the width of the continental shelf, because the action of the wind with regard to the surface elevation is generally large in shallow water only.

To calculate ζ and S for this case, it is convenient to divide the wind traction T into the following two cases. Namely, let $t_1=\frac{a}{V}$ be the time elapsing till the front portion of the wind region reaches the coast after having generated at $x=a$. Then during $0<t<t_1$, the existence domain of T is $a-Vt<\lambda<a+2l-Vt$.

Similarly if $t_2-t_1=\frac{2l}{V}$ be the time elapsing till the whole of this region is ashore, then for during $t_2>t>t_1$ the existence domain of T will be $0<\lambda<2l-V(t-t_1)$.

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1. Detailed discussions concerning a similar problem will be found in Chap. III.

Now put $\int_{a-Vt}^{a+2l-V\tau} T\sin\alpha\lambda d\lambda \equiv T_1(\tau)$, and $\int_0^{2l-V(\tau-t_1)} T\sin\alpha\lambda d\lambda \equiv T_2(\tau)$, and denote the elevations due to the action T_1 as ζ_1 for $t<t_1$ and as ζ_2 for $t>t_1$, then

$$\zeta_1=\frac{2}{\pi c}\int_0^\infty \cos\alpha x d\alpha\int_0^t T_1\sin\alpha c(t-\tau)d\tau$$

$$=-\frac{2T}{\pi(c^2-V^2)}\int_0^\infty\left\{\cos\alpha(a+2l-Vt)-\cos\alpha(a-Vt)\right\}\frac{\cos\alpha x}{\alpha^2}d\alpha$$

$$+\frac{T}{\pi c(c+V)}\int_0^\infty\left\{\cos\alpha(a+2l+ct)-\cos\alpha(a+ct)\right\}\frac{\cos\alpha x}{\alpha^2}d\alpha$$

$$-\frac{T}{\pi c(c-V)}\int_0^\infty\left\{\cos\alpha(a+2l-ct)-\cos\alpha(a-ct)\right\}\frac{\cos\alpha x}{\alpha^2}d\alpha,$$

$$\zeta_2=\frac{2}{\pi c}\int_0^\infty\cos\alpha x d\alpha\int_0^{t_1} T_1\sin\alpha c(t-\tau)d\tau=\frac{2T}{\pi(c^2-V^2)}\int_0^\infty\frac{\cos\alpha c(t-t_1)\cos\alpha x}{\alpha^2}d\alpha$$

$$-\frac{T}{\pi c(c+V)}\int_0^\infty\left\{\cos\alpha(a+ct)+\cos\alpha(2l+ct-t_1)-\cos\alpha(a+2l+ct)\right\}\times$$

$$\frac{\cos\alpha x}{\alpha^2}d\alpha-\frac{T}{\pi c(c-V)}\int_0^\infty\left\{\cos\alpha(a-ct)+\cos\alpha(2l-ct-t_1)\right.$$

$$\left.-\cos\alpha(a+2l-ct)\right\}\frac{\cos\alpha x}{\alpha^2}d\alpha.$$

Next also denote the elevations due to the action T_2 as ζ_3 for $t_1<t<t_2$ and as ζ_4 for $t>t_2$, then

$$\zeta_3=\frac{2}{\pi c}\int_0^\infty\cos\alpha x d\alpha\int_{t_1}^t T_2\sin\alpha c(t-\tau)d\tau$$

$$=\frac{-2T}{\pi c^2}\int_0^\infty\left\{\cos\alpha c(t-t_1)-1\right\}\frac{\cos\alpha x}{\alpha^2}d\alpha-\frac{2T}{\pi(c^2-V^2)}\times$$

$$\int_0^\infty\frac{\cos\alpha(2l+a-Vt)\cos\alpha x}{\alpha^2}d\alpha+\frac{T}{\pi c(c+V)}\int_0^\infty\frac{\cos\alpha(2l+ct-t_1)\cos\alpha x}{\alpha^2}d\alpha$$

$$+\frac{T}{\pi c(c-V)}\int_0^\infty\frac{\cos\alpha(2l-ct-t_1)\cos\alpha x}{\alpha^2}d\alpha,$$

$$\zeta_4=\frac{2}{\pi c}\int_0^\infty\cos\alpha x d\alpha\int_{t_1}^{t_2} T_2\sin\alpha c(t-\tau)d\tau$$

$$=-\frac{2T}{\pi c^2}\int_0^\infty\left\{\cos\alpha c(t-t_1)-\cos\alpha c(t-t_2)\right\}\frac{\cos\alpha x}{\alpha^2}d\alpha$$

$$-\frac{2T}{\pi(c^2-V^2)}\int_0^\infty\frac{\cos\alpha c(t-t_2)\cos\alpha x}{\alpha^2}d\alpha+\frac{T}{\pi c(c-V)}\times$$

$$\int_0^\infty\frac{\cos\alpha(2l+ct-t_1)\cos\alpha x}{\alpha^2}d\alpha+\frac{T}{\pi c(c+V)}\int_0^\infty\frac{\cos\alpha(2l-ct-t_1)\cos\alpha x}{\alpha^2}d\alpha,$$

by making suitable combinations of ζ_1, ζ_2, ζ_3 and ζ_4, the required elevation becomes

$$\left.\begin{array}{ll} \zeta=\zeta_1 & \text{for } 0<t<t_1 \\ \zeta=\zeta_2+\zeta_3 & \text{for } t_1<t<t_2 \\ \zeta=\zeta_2+\zeta_4 & \text{for } t>t_2 \end{array}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(12).$$

Now, put $x=0$ in the above formulae, then we have the time variation of the surface elevation at the coast. But there may be several different cases according to the relative magnitude of the free wave velocity (c) and the travelling velocity (V) of the wind region, i. e.,

$$\text{(a)}\quad \frac{a}{c}<\frac{a+2l}{c}<\frac{a}{V}, \qquad \text{(b)}\quad \frac{a}{c}<\frac{a}{V}<\frac{a+2l}{c},$$

and the cases of $\frac{a}{V}<\frac{a}{c}$. But as generally $V<c$, we will not concern for the last cases.

Now, integrate them by referring to a well known formula $\left(\int_0^\infty \frac{\cos bx}{x^2}dx = -\frac{\pi}{2}b \text{ for } b>0\right)$, then we have the following results.

For the case $\frac{a}{c}<\frac{a+2l}{c}<\frac{a}{V}$

$$\left.\begin{array}{ll} \zeta=0 & \text{for } t<\frac{a}{c} \\ \zeta=\frac{T(ct-a)}{c(c-V)} & \text{for } \frac{a}{c}<t<\frac{a+2l}{c} \\ \zeta=\frac{2Tl}{c(c-V)} & \text{for } \frac{a+2l}{c}<t<\frac{a}{V} \\ \zeta=\frac{T(a+2l-Vt)}{c(c-V)} & \text{for } \frac{a}{V}<t<\frac{a+2l}{V} \\ \zeta=0 & \text{for } \frac{a+2l}{V}<t \end{array}\right\} \quad \cdots\cdots(13'),$$

and for the case $\frac{a}{c}<\frac{a}{V}<\frac{a+2l}{c}$

$$\left.\begin{array}{ll} \zeta=0 & \text{for } t<\frac{a}{c} \\ \zeta=\frac{T(ct-a)}{c(c-V)} & \text{for } \frac{a}{c}<t<\frac{a}{V} \\ \zeta=\frac{Tct}{c^2} & \text{for } \frac{a}{V}<t<\frac{a+2l}{c} \end{array}\right\} \quad \cdots\cdots(13''),$$

$$\left.\begin{aligned} \zeta &= \frac{T(a+2l-Vt)}{c(c-V)} && \text{for } \frac{a+2l}{c} < t < \frac{a+2l}{V} \\ \zeta &= 0 && \text{for } t > \frac{a+2l}{V} \end{aligned}\right\}$$

and as the extreme case if $V=c$

$$\begin{aligned} \zeta &= 0 && \text{for } t < \frac{a}{c} \\ \zeta &= \frac{T}{c^2}ct && \text{for } \frac{a}{c} < t < \frac{a+2l}{c} \\ \zeta &= 0 && \text{for } t > \frac{a+2l}{c}. \end{aligned}$$

Now, we will give the more detailed interpretations about the development of the wave. We may obtain the following diagrams which express the relation between ζ and V, and that between V and the time (t_0) to attain the maximum wave height.

Fig. 2a

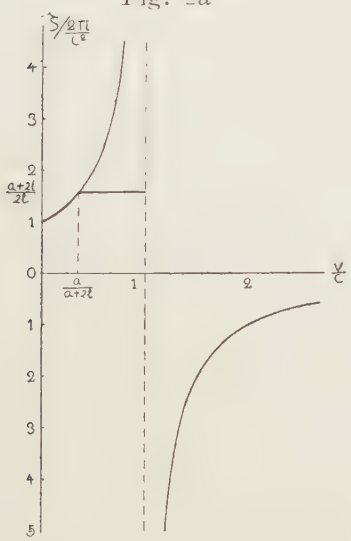
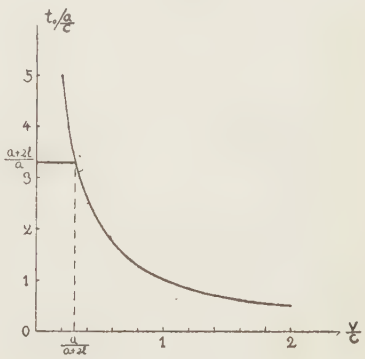


Fig. 2b



Referring to these, we know that if $V < \frac{a}{a+2l}c$, ζ increases with V approaching to c, but when $V \geqq \frac{a}{a+2l}c$, ζ takes a constant value $\frac{T(a+2l)}{c^2}$ (this is a maximum value due to a resonance effect) and increases no more even if V approaches to c still more, and untill

$V<\frac{a}{a+2l}c$, t_0 remains at a constant value $\frac{a+2l}{c}$ but for $V>\frac{a}{a+2l}c$, t_0 decreases inversely proportional to V and if V is equal to c it becomes $\frac{a}{c}$. Thus for an initially motionless sea, ζ can not increase infinitely (unless the width of the continental shelf a is very large), as generally considered, even if V is equal to c. If we don't consider the initial condition and treat the problem as quasisteady, there may exist a solution of ζ which gives an infinite value for $V=c$, yet it must correspond to the case where the width of the continental shelf a is infinitely large or t_0 is infinitely great. But, as such assumption is very far from the actuality, it may be said that the height of wave at the real coast can not be so large even by the resonance effect.

(*b*). *The case in which the region travels from the coast seawards*

We shall consider the case of the region having the width $2l$ beginning to advance from the coast seawards and dying away when its tail end reaches a point at a distance a from the coast (a is taken as the breadth of the continental shelf).

As in the preceding case it is convenient to divide the wind action T into the following two parts. Let $t_1=\frac{2l}{V}$ be the time elapsing before the whole region appears over the sea-surface after its front has reached the coast, and let $t_2-t_1=\frac{a}{V}$ be the time elapsing between the region's first appearance over the sea-surface and its almost total loss of action.

Then the existence domain of the wind traction T is obviously $0<\lambda<Vt$ for $0<t<t_1$, and $V(t-t_1)<\lambda<Vt$ for $t_1<t<t_2$.

Now put $\int_0^{V\tau} T\sin\alpha\lambda d\lambda\equiv T_1(\tau)$, and $\int_{V(\tau-t_1)}^{V\tau} T\sin\alpha\lambda d\lambda\equiv T_2(\tau)$, and let the elevations due to the action T_1 be ζ_1 for $t<t_1$, and ζ_2 for $t>t_1$, then

$$\zeta_1=\frac{2}{\pi c}\int_0^\infty\cos\alpha x d\alpha\int_0^t T_1\sin\alpha c(t-\tau)d\tau=-\frac{2T}{\pi c^2}\int_0^\infty\frac{(\cos\alpha ct-1)\cos\alpha x}{\alpha^2}d\alpha$$

$$-\frac{2T}{\pi(c^2-V^2)}\int_0^\infty\frac{(\cos\alpha Vt-\cos\alpha ct)\cos\alpha x}{\alpha^2}d\alpha,$$

$$\zeta_2=\frac{2}{\pi c}\int_0^\infty\cos\alpha x d\alpha\int_0^{t_1} T_1\sin\alpha c(t-\tau)d\tau$$

$$= -\frac{2T}{\pi c^2}\int_0^\infty \frac{\{\cos \alpha ct - \cos \alpha c(t-t_1)\}\cos \alpha x}{\alpha^2} d\alpha + \frac{2T}{\pi(c^2-V^2)} \times$$
$$\int_0^\infty \frac{\cos \alpha ct \cos \alpha x}{\alpha^2} d\alpha - \frac{T}{\pi c(c-V)}\int_0^\infty \frac{\cos \alpha(ct - c-Vt_1)\cos \alpha x}{\alpha^2} d\alpha$$
$$-\frac{T}{\pi c(c-V)}\int_0^\infty \frac{\cos \alpha(ct - c+Vt_1)\cos \alpha x}{\alpha^2} d\alpha.$$

Also let the elevations due to the action T_2 be ζ_3 for $t_1 < t < t_2$ and ζ_4 for $t > t_2$, then

$$\zeta_3 = \frac{2}{\pi c}\int_0^\infty \cos \alpha x\, d\alpha \int_{t_1}^t T_2 \sin \alpha c(t-\tau) d\tau = -\frac{2T}{\pi(c^2-V^2)} \times$$
$$\int_0^\infty \left\{\cos \alpha Vt - \cos \alpha V(t-t_1) + \cos \alpha c(t-t_1)\right\}\frac{\cos \alpha x}{\alpha^2} d\alpha + \frac{T}{\pi c(c-V)} \times$$
$$\int_0^\infty \frac{\cos \alpha(ct - c-Vt_1)\cos \alpha x}{\alpha^2} d\alpha + \frac{T}{\pi c(c+V)}\int_0^\infty \frac{\cos \alpha(ct - c+Vt_1)\cos \alpha x}{\alpha^2} d\alpha,$$

$$\zeta_4 = \frac{2}{\pi c}\int_0^\infty \cos \alpha x\, d\alpha \int_{t_1}^{t_2} T_2 \sin \alpha c(t-\tau) d\tau = -\frac{2T}{\pi(c^2-V^2)} \times$$
$$\int_0^\infty \frac{\cos \alpha c(t-t_1)\cos \alpha x}{\alpha^2} d\alpha - \frac{T}{\pi c(c+V)}\int_0^\infty \left\{\cos \alpha(a+2l-c\overline{t-t_2})\right.$$
$$\left. -\cos \alpha(2l-c\overline{t-t_1}) - \cos \alpha(a-c\overline{t-t_2})\right\}\frac{\cos \alpha x}{\alpha^2} d\alpha - \frac{T}{\pi c(c-V)} \times$$
$$\int_0^\infty \left\{\cos \alpha(a+2l+c\overline{t-t_2}) - \cos \alpha(2l+c\overline{t-t_1}) - \cos \alpha(a+c\overline{t-t_2})\right\}\frac{\cos \alpha x}{\alpha^2} d\alpha.$$

By making suitable combinations of ζ_1, ζ_2, ζ_3 and ζ_4, we can get

$$\left.\begin{array}{ll} \zeta = \zeta_1 & \text{for } t < t_1 \\ \zeta = \zeta_2 + \zeta_3 & \text{for } t_1 < t < t_2 \\ \zeta = \zeta_2 + \zeta_4 & \text{for } t > t_2 \end{array}\right\} \quad \ldots\ldots\ldots\ldots(14).$$

Then time relation of the surface elevation at the coast may be given by

$$\left.\begin{array}{ll} \zeta = \dfrac{Vt}{c(c+V)}T & \text{for } t < \dfrac{2l}{V} \\ \zeta = \dfrac{T.2l}{c(c+V)} & \text{for } \dfrac{2l}{V} < t < \dfrac{a}{c} + \dfrac{a+2l}{V} \\ \zeta = \dfrac{T(a+2l \quad c\overline{t \quad t_2})}{c(c+V)} & \text{for } \dfrac{a}{c} + \dfrac{a+2l}{V} < t < \left(\dfrac{1}{V} + \dfrac{1}{c}\right)(a+2l) \\ \zeta = 0 & \text{for } t > \left(\dfrac{1}{V} + \dfrac{1}{c}\right)(a+2l) \end{array}\right\}$$
$$\ldots\ldots\ldots\ldots(14').$$

As the maximum height of the elevation at the coast is given by $\zeta = \dfrac{T.2l}{c(c+V)}$, ζ decreases with V approaching to c in opposition to the

preceding case (a). Namely *in this case the so-called resonance effect does not occur at all*, and the elevation of the water surface at the coast is smaller than that of the case (a) if the other conditions are the same. And the ratio of the elevations for the above two cases is given by $\frac{c+V}{c-V}$ if $\frac{a}{c}<\frac{a+2l}{c}<\frac{a}{V}$, or by $\frac{(a+2l)(c+V)}{2cl}$ if $\frac{a}{c}<\frac{a}{V}<\frac{a+2l}{c}$.

J. Proudman, in the investigations about the effect of the travelling pressure disturbance over the sea, has obtained the theoretical result that the ratio of the wave heights in the above two cases is given by $\frac{c+V}{c-V}$, and this form holds for the all possible value of V. But in the present investigation (about the effect of the travelling local gale) this form holds only when $\frac{a}{c}<\frac{a+2l}{c}<\frac{a}{V}$ as above stated.

Now, we will give the numerical explanation about the result now obtained. We have known already that ζ takes the form (13′) or (13″) according to $\frac{c}{V}$ being greater or smaller than $\frac{a+2l}{a}$. But along the coast of the Japanese island (for an example) the widths of the continental shelves a are generally 20 km~35 km and the length of the wind region may be considered to be about 40 km~60 km, therefore $\frac{a+2l}{a}$ (it may be called the critical value of resonance from its physical meaning) takes the value of 2~4.

Put $a=30$ km, $2l=50$ km, wind velocity $W=30$ m/sec and $h=50$ m, then $\frac{a+2l}{a}\fallingdotseq 2.7$, $T=0.0025\cdot 0.00129\cdot(30\cdot 10^2)^2$ c.g.s.$\fallingdotseq 30$ C.G.S. and $c^2=gh=5\cdot 10^6$c.g.s. or $c=80$ km/hour. If $V<\frac{c}{2.8}$ or $V<30$km/hour the surface elevation in the case (a) is given by $\zeta=\frac{24}{\left(1-\frac{V}{c}\right)}$cm, and ζ increases with V; but when V exceeds $\frac{c}{2.7}$ i. e., V is over 30 km/hour, then from (13″) the elevation takes a constant value 42cm and increases no more even if V becomes equal to c. In the case (b),

ζ is given by $\frac{24}{\left(1+\frac{V}{c}\right)}$cm. Supposing $V=\frac{c}{2.7}$, then ζ is 17cm and it is about $\frac{1}{2.5}$ of that of the case (a).

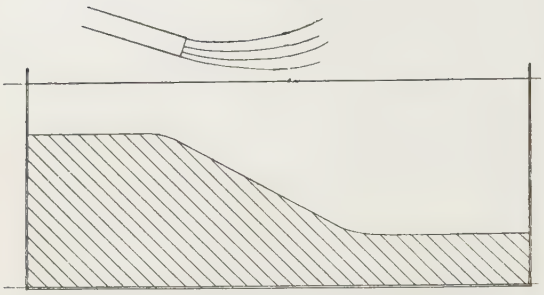
III. Internal Waves of Meteorological Origin

The sea occasionary consists of two horizontal layers, whose densities are different from one another but may be considered uniform within each layer. On the boundary surface of these two layers the phenomena of the so-called internal wave of great height are often observed. These peculiar phenomena were first observed by Dr. Helland-Hansen and Dr. Nansen, namely, during the North Polar expedition from May to July, 1894. They observed that, at a depth 200–300 m in the sea, the density was discontinuous, and that the transition layer was rising and falling to the extent of about 20–30 metres. After then W. J. Sandström endeavoured to explain experimentally the mechanism of this peculiar phenomenon. Fig. 4 shows the vertical section of the water-tank used for the experiment, in which the upper layer is light water and the lower one is heavy water, and is coloured to make the boundary surface distinct. By letting a strong, narrow air-current suddenly act locally, at a certain place on the surface of the light water-layer, this layer was put in motion in the direction of the air-current, and the underlying heavier water at this place and behind it, was lifted to form a great wave, while in front of it the heavier water layer was depressed. If the air-current continues to act on the surface at the same spot for some time, the wave in the underlying heavier water will remain permanent, altering its shape gradually untill new condition of stable equilibrium is attained. If the spot at which the air-current acts on the water-surface moves forward (in the direction of the air-current), the wave will follow in the same direction.

If the air-current suddenly stops, the wave will continue its course as a boundary-wave, untill it reaches the wall of the vessel, or if we think of the sea, untill it reaches the side-slope of the sea-basin. Here it will be partly broken and reflected, and will again pass across the basin, till it reaches the other side.

In the same manner a sudden storm or wind on a sea affects the water strata and may produce a boundary wave of great height. This explanation is an outline of Sandström's experiment, and the present

Fig. 3



Boundary-Wave formed by Local Air-Current (cf. Sandström)

writer will proceed from a theoretical standpoint with his discussion of these phenomena, and especially of the aspect of the development and the propagation of waves produced by a sudden storm or wind.

Now before proceeding with our discussion, we shall make the following assumptions, namely that the ocean consists of two layers whose densities are different through uniform in each layer, that there is no friction at the bottom and on the boundary surface, and that the problem may be considered as of two dimensions.

Now let us denote the density, the thickness, the surface elevation and the current velocity of x-direction in the upper and the lower strata by (ρ_1, ρ_2), (h_1, h_2), (ζ_1, ζ_2) and (u_1, u_2) respectively, then by neglecting the small terms, the equation of motion and the equation of continuity become

$$\rho_1\frac{\partial u_1}{\partial t}=\mu_1\frac{\partial^2 u_1}{\partial z^2}-g\rho_1\frac{\partial \zeta_1}{\partial x} \qquad \text{for upper layer} \quad \ldots(15),$$

$$\rho_2\frac{\partial u_2}{\partial t}=\mu_2\frac{\partial^2 u_2}{\partial z^2}-g\rho_1\frac{\partial \zeta_1}{\partial x}-g(\rho_2-\rho_1)\frac{\partial \zeta_2}{\partial x} \qquad \text{for the lower layer} \quad \ldots(16),$$

$$\frac{\partial}{\partial x}\int_{-h_1}^{0}\rho_1 u_1 dz=-\rho_1\left(\frac{\partial \zeta_1}{\partial t}-\frac{\partial \zeta_2}{\partial t}\right) \quad \ldots(17),$$

$$\frac{\partial}{\partial x}\int_{-(h_1+h_2)}^{-h_1}\rho_2 u_2 dz=-\rho_2\frac{\partial \zeta_2}{\partial t} \quad \ldots(18).$$

The conditions at the upper surface, at the intermediate surface and at the bottom are respectively

$$\left.\begin{aligned} &\mu_1\frac{\partial u_1}{\partial z}=T && \text{for } z=0\\ &\mu_1\frac{\partial u_1}{\partial z}=0,\ \mu_2\frac{\partial u_2}{\partial z}=0 && \text{for } z=-h_1\\ &\mu_2\frac{\partial u_2}{\partial z}=0 && \text{for } z=-(h_1+h_2) \end{aligned}\right\} \quad \ldots(19),$$

and the initial condition is

$$u_1=u_2=0 \qquad \text{when } t=0 \qquad \text{.....(20).}$$

Now integrating (15) and (16) with regard to z in the intervals $(0,\ -h_1)$ and $(-h_1,-(h_1+h_2))$ respectively and putting $S_1\equiv\int_{-h_1}^{0}\rho_1 u_1\,dz$, and $S_2\equiv\int_{-(h_1+h_2)}^{-h_1}\rho_2 u_2\,dz$, then we have

$$\frac{\partial S_1}{\partial t}=T-g\rho_1 h_1\frac{\partial\zeta_1}{\partial x} \qquad \text{.....(15'),}$$

$$\frac{\partial S_2}{\partial t}=-g\rho_1 h_2\frac{\partial\zeta_1}{\partial x}-g(\rho_2-\rho_1)h_2\frac{\partial\zeta_2}{\partial x} \qquad \text{.....(16'),}$$

$$\frac{\partial S_1}{\partial x}=-\rho_1\left(\frac{\partial\zeta_1}{\partial t}-\frac{\partial\zeta_2}{\partial t}\right) \qquad \text{.....(17'),}$$

$$\frac{\partial S_2}{\partial x}=-\rho_2\frac{\partial\zeta_2}{\partial t} \qquad \text{.....(18').}$$

At first, let us assume that T is independent of t, then eliminating ζ_1, ζ_2, and S_2 from the above equations, we get

$$q\frac{\partial^4 S_1}{\partial t^4}-c^2(1+q)\frac{\partial^4 S_1}{\partial t^2\partial x^2}+c^4(1-p)\frac{\partial^4 S_1}{\partial x^4}=0 \qquad \text{...(21),}$$

where $q\equiv\dfrac{h_1}{h_2}$, $p\equiv\dfrac{\rho_1}{\rho_2}$, $c^2\equiv gh_1$.

Solve (21) under the initial conditions $S_1=S_2=\zeta_1=\zeta_2=0$ at $t=0$ or their equivalents

$$S_1=0,\ \frac{\partial S_1}{\partial t}=T,\ \frac{\partial^2 S_1}{\partial t^2}=0,\ \frac{\partial^3 S_1}{\partial t^3}=c^2\frac{\partial^2 T}{\partial x^2} \quad \text{when } t=0$$

$$\text{.....(22).}$$

Since eqs. (15′) to (18′) are obviously "additive" with respect to T in the meaning defined by Prof. T. Nomitsu, we can easily get the solution by his theorem[1] when T is variable with t. Then we have

$$S_1=\frac{c^2-c_2^2}{\pi(c_1^2-c_2^2)}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty}T\cos\alpha c_1(t-\tau)\cos\alpha(\lambda-x)d\lambda$$
$$+\frac{c_1^2-c^2}{\pi(c_1^2-c_2^2)}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty}T\cos\alpha c_2(t-\tau)\cos\alpha(\lambda-x)d\lambda \qquad \text{...(23),}$$

where $c_1^2=g(h_1+h_2)$, $c_2^2=\dfrac{g(\rho_2-\rho_1)h_1h_2}{\rho_2(h_1+h_2)}$,

$$\zeta_1=-\frac{c_1(c^2-c_2^2)}{\pi\rho_1c^2(c_1^2-c_2^2)}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty}T\sin\alpha c_1(t-\tau)\sin\alpha(\lambda-x)d\lambda$$
$$-\frac{c_2(c_1^2-c^2)}{\pi\rho_1c^2(c_1^2-c_2^2)}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty}T\sin\alpha c_2(t-\tau)\sin\alpha(\lambda-x)d\lambda \qquad \text{...(24),}$$

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1. Proc. Imp. Acad. Tokyo, **11**, 359 (1935).

$$\text{or}\quad \zeta_2=\frac{c_1(c^2-c_2^2)}{\pi\rho_1(c_1^2-c_2^2)}\left(\frac{1}{c^2}-\frac{1}{c_1^2}\right)\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty} T\sin\alpha c_1(t-\tau)\sin\alpha(\lambda-x)d\lambda$$
$$-\frac{c_2(c_1^2-c^2)}{\pi\rho_1(c_1^2-c_2^2)}\left(\frac{1}{c^2}-\frac{1}{c_2^2}\right)\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty} T\sin\alpha c_2(t-\tau)\sin\alpha(\lambda-x)d\lambda$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(25),$$

$$S_2=\frac{\rho_2(c^2-c_2^2)(c_1^2-c^2)}{\pi\rho_1c^2(c_1^2-c_2^2)}\int_0^t d\tau\int_0^\infty d\alpha\int_{-\infty}^{\infty} T\cos\alpha(\lambda-x)\times$$
$$\{\cos\alpha c_2(t-\tau)-\cos\alpha c_1(t-\tau)\}d\lambda \qquad \cdots\cdots\cdots\cdots(26).$$

Eqs. (24) and (25) will give, respectively, the elevations of the upper surface and of the lower intermediate surface. They contain two waves propagating with the external wave velocity c_1 and the internal wave velocity c_2.

Now, in order to obtain a concrete example, we shall consider the same wind region as that of Chap. II, namely $T=\text{const}$ for $-l<x-Vt<l$ and $T=0$ for $x-Vt>l$ and $x-Vt<-l$. Then

$$\zeta_1=\frac{c^2-c_2^2}{\pi\rho_1(c_1^2-c_2^2)c^2}\left\{\frac{2T}{1-\frac{V^2}{c_1^2}}\int_0^\infty\frac{\sin\alpha(x-Vt)\sin\alpha l}{\alpha^2}d\alpha-\frac{2T}{1-\frac{V}{c_1}}\times\right.$$
$$\left.\int_0^\infty\frac{\sin\alpha(x-c_1t)\sin\alpha l}{\alpha^2}d\alpha-\frac{T}{1+\frac{V}{c_1}}\int_0^\infty\frac{\sin\alpha(x+c_1t)\sin\alpha l}{\alpha^2}d\alpha\right\}$$
$$+\frac{c_1^2-c^2}{\pi\rho_1c^2(c_1^2-c_2^2)}\left\{\frac{2T}{1-\frac{V^2}{c_2^2}}\int_0^\infty\frac{\sin\alpha(x-Vt)\sin\alpha l}{\alpha^2}d\alpha-\frac{T}{1-\frac{V}{c_2}}\times\right.$$
$$\left.\int_0^\infty\frac{\sin\alpha(x-c_2t)\sin\alpha l}{\alpha^2}d\alpha-\frac{T}{1+\frac{V}{c_2}}\int_0^\infty\frac{\sin\alpha(x+c_2t)\sin\alpha l}{\alpha^2}d\alpha\right\}\ldots(27),$$

$$\zeta_2=\frac{c^2-c_2^2}{\pi\rho_1(c_1^2-c_2^2)}\left(\frac{1}{c^2}-\frac{1}{c_1^2}\right)\left\{\frac{2T}{1-\frac{V^2}{c_1^2}}\int_0^\infty\frac{\sin\alpha(x-Vt)\sin\alpha l}{\alpha^2}d\alpha\right.$$
$$\left.-\frac{T}{1-\frac{V}{c_1}}\int_0^\infty\frac{\sin\alpha(x-c_1t)\sin\alpha l}{\alpha^2}d\alpha-\frac{T}{1+\frac{V^2}{c_1^2}}\int_0^\infty\frac{\sin\alpha(x+c_1t)\sin\alpha l}{\alpha^2}d\alpha\right\}$$
$$+\frac{c_1^2-c^2}{\pi\rho_1(c_1^2-c_2^2)}\left(\frac{1}{c^2}-\frac{1}{c_2^2}\right)\left\{\frac{2T}{1-\frac{V^2}{c_2^2}}\int_0^\infty\frac{\sin\alpha(x-Vt)\sin\alpha l}{\alpha^2}d\alpha\right.$$
$$\left.-\frac{T}{1-\frac{V}{c_2}}\int_0^\infty\frac{\sin\alpha(x-c_2t)\sin\alpha l}{\alpha}d\alpha-\frac{T}{1+\frac{V}{c_2}}\int_0^\infty\frac{\sin\alpha(x+c_2t)\sin\alpha l}{\alpha^2}d\alpha\right\}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(27').$$

Now, if, for instance, we put $\rho_1=1.0275$, $\rho_2-\rho_1=0.0005$, $h_1=100$ m, $h_2=200$ m and $g=9.8$ m/sec, we get $c_1=450$ m/sec and $c_2=0.68$ m/sec. Therefore $\frac{c^2}{c_1^2}=5.10^{-3}$, $\frac{c_2^2}{c_1^2}=3.10^{-6}$, $\frac{c_2^2}{c^2}=5.10^{-4}$ are very small and may be neglected in comparison with unity. Moreover the first two terms become large when V approaches to c_1 very closely, but since here c_1 is 450 m/sec and the value of V, even if it is estimated very highly, is about 30 m/sec, in this deep sea V cannot actually approach to c_1, and consequently the first three terms of ζ_2 (two free waves of the velocity c_1 and the forced wave of the velocity V) are very small. However, as c_2 is proportional to $(\rho_2-\rho_1)^{\frac{1}{2}}$, the terms containing c_2 in denominators are very large compared with the other terms. Therefore, neglecting the small terms, the main features of the internal waves may be given by the following forms.

$$\zeta_2=\frac{T}{\pi\rho_1 c_2^2}\left\{\frac{1}{1-\frac{V}{c_2}}\int_0^\infty\frac{\sin\alpha(x-c_2t)\sin\alpha l}{\alpha^2}d\alpha+\frac{1}{1+\frac{V}{c_2}}\times\right.$$
$$\left.\int_0^\infty\frac{\sin\alpha(x+c_2t)\sin\alpha l}{\alpha^2}d\alpha-\frac{2}{1-\frac{V^2}{c_2^2}}\int_0^\infty\frac{\sin\alpha(x-Vt)\sin\alpha l}{\alpha^2}d\alpha\right. \qquad \ldots\ldots(28').$$

This formula is similar to (9'), therefore it may be written as follows :—

$$\left.\begin{array}{lll}
\text{1st term}=\dfrac{T}{2(c_2-V)c_2\rho_1}(x-c_2t) & \text{for} & -l<x-c_2t<l \\
\text{,,}\quad=\dfrac{\pm T}{2(c_2-V)c_2\rho_1}l & \text{for} & x-c_2t>l \text{ and } x-c_2t<-l \\
\text{2nd term}=\dfrac{T}{2(c_2+V)c_2\rho_1}(x+c_2t) & \text{for} & -l<x+c_2t<l \\
\text{,,}\quad=\dfrac{\pm T}{2(c_2+V)c_2\rho_1}l & \text{for} & x+c_2t>l \text{ and } x+c_2t<-l \\
\text{3rd term}=\dfrac{T}{(c_2^2-V^2)\rho_1}(x-Vt) & \text{for} & -l<x-Vt<l \\
\text{,,}\quad=\dfrac{\mp T}{(c_2^2-V^2)\rho_1}l & \text{for} & x-Vt>l \text{ and } x-Vt<-l
\end{array}\right\} \qquad \ldots\ldots(28'').$$

The above formulae are expressed diagramatically in Fig. 4a, 4b and 4c, where the three cases $V=0$ and $V\gtrless c_2$ are given, but among these the cases $V<c_2$ and $V=0$ seem to correspond to those of the Sandström experiment. However, since the travelling velocity of the

Fig. 4a.

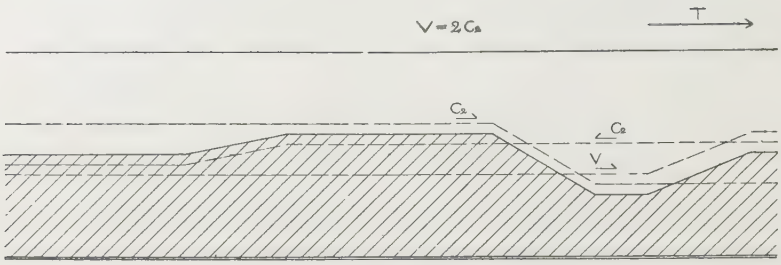


Fig. 4b.

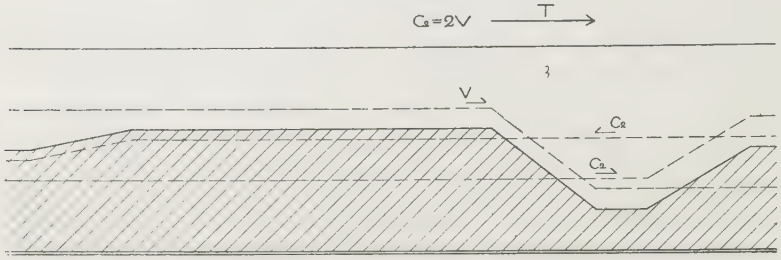
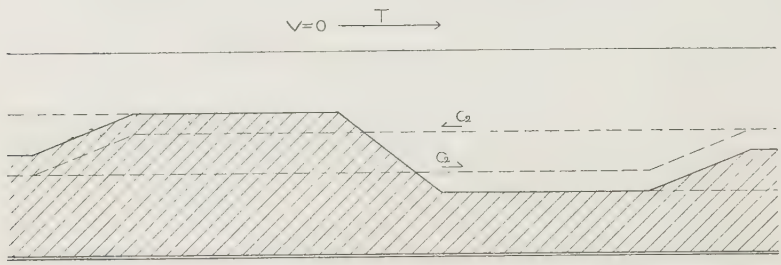


Fig. 4c.



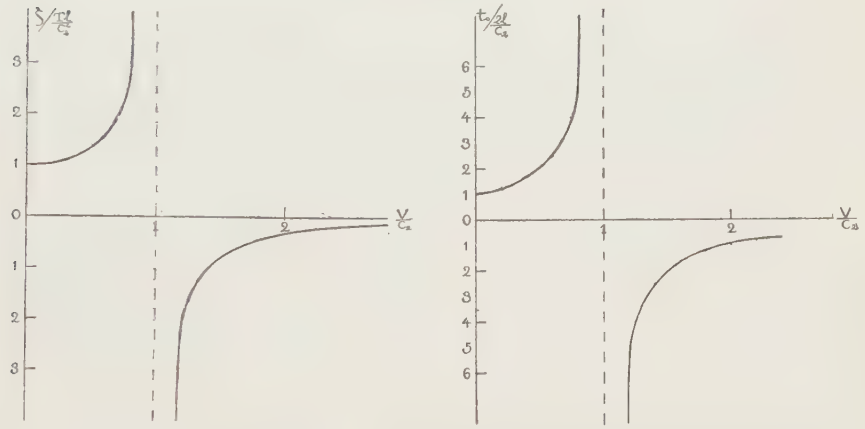
wind region is generally greater than the internal wave velocity c_2, the circumstances are not generally like those of the Sandström experiment.

Now let us discuss the results of (28′) in detail. The maximum height of the wave is given by $\zeta_2=\dfrac{Tl}{(c_2^2-V^2)\rho_1}$, where $c_2=\left(\dfrac{g(\rho_2-\rho_1)h_1h_2}{\rho_2(h_1+h_2)}\right)^{\frac{1}{2}}$ and it is about 1 m/sec for a deep sea such as here considered. And since the time that the wave height becomes to maximum is given by $t_0=\dfrac{2l}{c_2-V}$ and it is related to the travelling velocity V like that of ζ_2, to discuss the development of the large wave heights we must, in addition to the values T and l, take into account the effect of the value V for them.

For the above purpose, in Fig. 5a and 5b we shall give the relations

of $\frac{V}{c_2}$ to ζ_2 or t_0, where $\frac{\zeta}{Tl/c_2^2}$ and $\frac{t_0}{2l/c_2}$ are taken as the ordinate and $\frac{V}{c_2}$ as the abscissa. Now if, as a very usual example, we take $l=20$ km, $c_2=0.68$ m/sec, wind velocity $W=20$ m/sec and use $T=0.0025\,\rho_{air}W^2$, then we get $\frac{Tl}{c_2^2}\fallingdotseq 40$ m and $t_0\fallingdotseq 16$ hours.

Fig. 5a and 5b.



By referring to the above two figures we obtain the following results, namely that the waves usually observed (about 40 metres in height) correspond to the neighbourhood $V=0$, that, for the generation of such a large wave of about 200 m height, the wind velocity and the area of the wind region must be, of course, very large, but also it is necessary that the wind region travelling with a velocity nearly equal to c_2 continues to act for a long time and consequently the so-called resonance phenomena occurs, and on the other hand that, if the travelling velocity of the wind region is much larger than c_2, the wave cannot become large even if the wind velocity and the width of the region are great.

From the above discussion it has become clear that large internal waves can be produced only under suitable conditions as above stated, even under the effect of a local gale of large scale.

We shall next touch the effect of the pressure disturbance for the development of the internal wave briefly. It is clear that the formulae giving the elevations of the upper surface and of the lower boundary surface under the effect of the travelling atmospheric pressure disturbance are

$$\zeta_1=\frac{c_1}{2}\int_0^t\left\{\gamma(x-c_1t+\overline{c_1-V}\tau)-\gamma(x+c_1t-\overline{c_1+V}\tau)\right\}d\tau,$$

$$\zeta_2=\frac{c_2^2-c_1^2}{2c_1}\int_0^t\left\{\gamma(x+c_1t-\overline{c_1+V}\tau)-\gamma(x-c_1t+\overline{c_1-V}\tau)\right\}d\tau,$$

where γ is $\dfrac{\partial\bar{\zeta}}{\partial x}$ and $\bar{\zeta}$ is the atmospheric pressure measured by the water column.

Now it must be noticed that, since the above ζ_1 and ζ_2 contain no terms inversely proportional to $\rho_2-\rho_1$, the *waves cannot be developed greatly by pressure disturbance as by the wind action*, and also that, since the wave velocity is in all cases the external free wave velocity and is not the internal one, in such a deep sea as that under consideration the so-called *resonance phenomena due to the travelling pressure disturbance can scarcely occur in practice.*

Therefore the great internal wave cannot develop from the pressure disturbance, even if the travelling effect is considered. The above results have been deduced from the most idealized case, but we infer the general features about the development of the internal boundary waves of the large height from them.

In conclusion the author wishes to express his sincere thanks to Prof. T. Nomitsu for his kind advices and encouragement during the study.

Characteristics of the Activity and the Peculiar Product "SARA-ISI" of the Aso Volcano

By Munetosi Namba

(Received March 20, 1936)

Abstract

There is a curious stone called "SARA-ISI" among the products from the Volcano Aso. A typical Sara-isi is a somewhat flat rock fragment, the margin of which is rimmed by a projection just like the margin of a dish. Hitherto, the origin of the rim was attributed to the impulsive action which took place when a plastic magma or new magma covered a pre-existing rock dashed against the ash field.

The writer made many year's investigation of the place and manner of existense and the constraction of Sara-isi. The results are as follows:—

1. The writer discriminated the upper and lower sides of a Sara-isi reasonably.

2. About the genesis of Sara-isi, the writer was obliged to modify previous illustriations, and arrived at the conclusion that the rim of a Sara-isi is a secondary or posterior product which is always growing, and that it is not produced accidentally by the volcanic outburst.

3. The alternative eruptions and states of hot water pool being constantly repeated in the case of Aso account for the special production of Sara-isi. The writer calls shortly such a volcanic activity as "Aso-type."

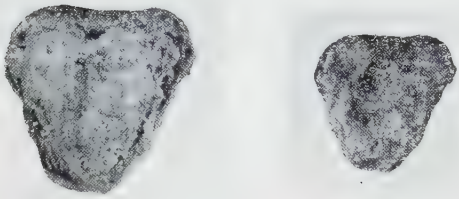
4. The Naka-dake craters have continued their actions of Aso-type since prehistorical time. Kisima-dake made also the Aso-type volcanic activity in pre-historic ages, and this volcano is younger than volcano Ô-jo-dake.

§ 1. Introduction

Among the various products from volcano Aso, there is a curious kind of stones called "Sara-isi" (meaning dish-stone). Sara-isi is a somewhat flat rock fragment. It was named "Lava-cake" by Prof. Dr. T. Hiki on 1914[1] and commonly has a plate-like suspension along its margin as the eaves of a house, but sometimes two or more parallel suspensions are often produced. These suspensions are commonly called "the rim of Sara-isi" (Fig. 1, 2, 3, 4, 5, 6). Since we have no report about ejecta like Sara-isi except from Aso, we may say surely that they might not exist in other places in Japan and likely not in other countries. If this is so, then this peculiar production of Sara-isi must correspond to the characteristics of the activity of Aso-volcano, and the study of Sara-isi must surely become a key to the

1. Catalogue of Minerals and Rocks from Shimadzu Seisaku-sho Ltd. 1914.

Fig. 1. Sara-isi with one rim.



Lower-side Upper-side

Fig. 3. Sara-isi of flat type.

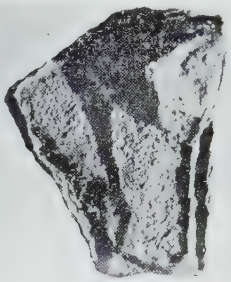
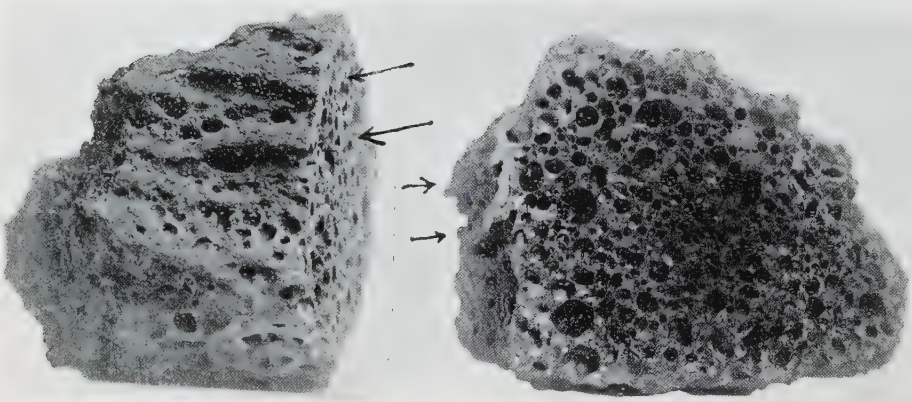


Fig. 2. Sara-isi with two rims.

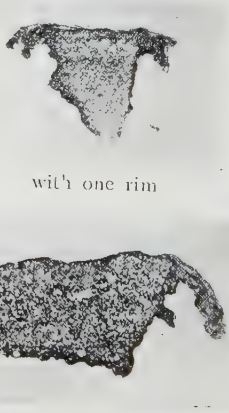


Side Section

Fig. 4. Sara-isi of irregular type.

Stalactitic processes

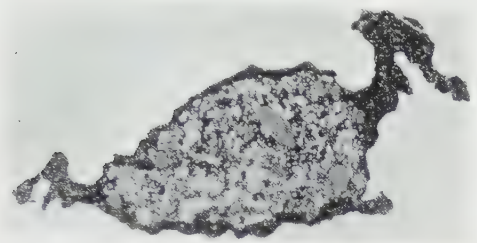
Fig. 5. Sections.



with one rim

with two rims

Fig. 6. Sections.

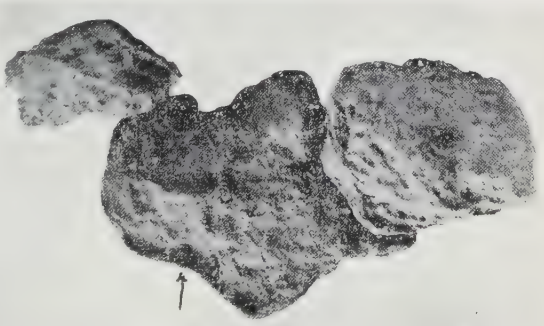


Irregular Sara-isi

Flat Sara-isi

elucidation of the peculiarity of the volcanic activity of Aso. Sara-isi was heretofore collected only at Sara-yama, a part of the ancient encircling crater-rim of Naka-dake (the only one active central cone among the great Aso cones), but the writer, prospecting for several years, discovered Sara-isi not only around the ancient crater of Naka-dake (north, west and south parts), but also in the neighbourhood of the Kisima-dake crater and especially at Ô-jo-dake (both are the central cones of Aso). (Fig. 7, 8, 9). It is generally described in the Japanese text books of geology as a curious kind of volcanic bomb or lapilli.

Fig. 7. Sara-isi from Ô-jo-dake.



Heretofore published opinions about the method of production of Sara-isi may be summarized as follows :—

Mr. Sato[1] and Mr. Igi[2] say that Sara-isi is a volcanic ejectamenta. Some magma once were hurled up into the atmosphere and fell down on the flat and soft ash-field, then it was depressed in the central part and contrarily bent upwards at its margin so that a flat dish-like block of

1. 佐藤傳藏 :—阿蘇山の皿石, 地質學雜誌 **2**, 356 (1895).
2. 伊木常誠 :—阿蘇火山調査報文, 震災豫防調査會報告 **33**, 77 (1901).

lava (Sara-isi) was then formed. Examining its section, therefore, it may be understood that near the surface its structure is compact and rather more homogeneous to the naked eyes than the inner part which is rather roughly granulated . . . . . etc.

In spite of such illustrations, any one who examines many Sara-isi carefully, will at once find that the central part is not always the

Fig. 8. Sara-isi from Kisima-dake.

newly erupted magma block, but is often the pre-existing rock fragment. This fact shows that some main bodies of Sara-isi cannot be said to be conclusively illustrated in the above mentioned way.

In the next place, Prof. Z. Harada[1] gave his opinion as follows :—

1. The upper surface of each block (he means the outside of the dish in the popular sense) is rough and ragged, while the lower inside of the dish is compact and smooth, and has many wart-like low prominences just like those on the upper surface of a semisolid substance which is dashed on a floor. . . . . .

2. Sometimes, the colour differs in different parts of one block, e. g. lighter or more bleached on the upper side than on the lower.

3. Sara-isi encloses always a foreign rock-material, and the enclosure is sometimes a piece of pre-existing rock, in other cases a lapillii ; while in still other cases a fragment of tuff or a lump of

1. Z. Harada :—"Sara-isi" or Dish-Stone of the Volcano-ASO. Proc. of Imp. Academy **3** 539 (1927). 原田準平:—阿蘇之皿石, 地質學雜誌 **39**, 736 (1927).

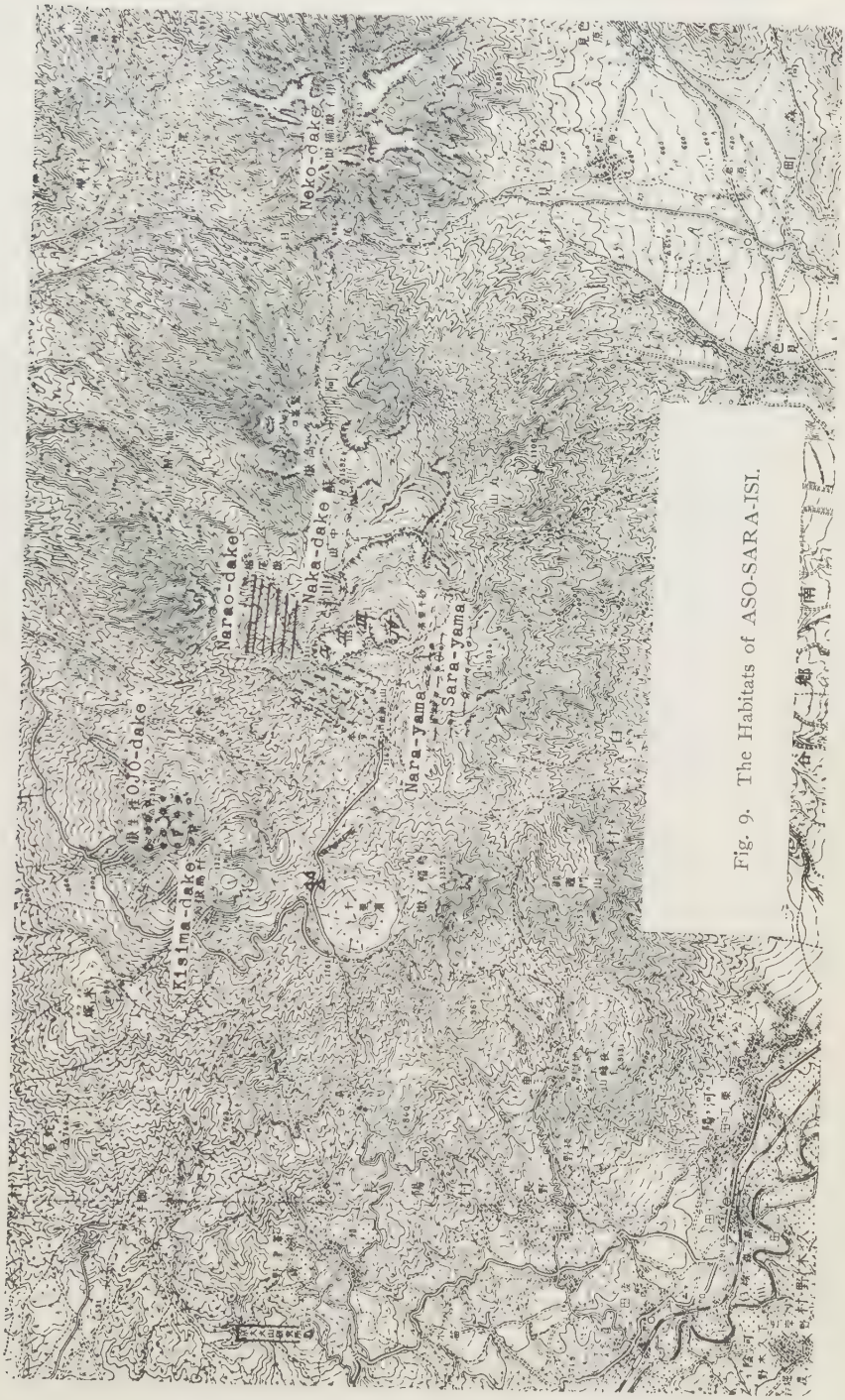


Fig. 9. The Habitats of ASO-SARA-ISI.

volcanic ash. The crust of Sara-isi is thin, usually less than 1 m.m. in thickness, and can be easily taken off in some cases.

4. The crust is thicker on the lower side of each block than on the upper. Some lumps of volcanic ash have the crusts only on the lower side.

5. The rim at the margin of the crust of each block is mostly upward projecting, while sometimes it is projecting on both the upper and lower sides; in other cases it is irregular and discontinuous, and in still other cases the rim turns in the opposite directions at the opposite ends.

6. The crust of Sara-isi is black or dark gray and aphanitic. Under the microscope, it is partly vitreous with a fluxion texture, showing flow-bands around the core, and partly of a vitroclastic tufaceous character, composed of fragmented pieces of pyroxenes and calcic plagioclases with a small quantity of cusp-shaped glass and corroded rock fragments, cemented by brown basic glass.

7. The above facts show that the crust of Sara-isi was formed by mingling of fluidal lava material with fine volcanic ashes, i. e. the crust is made of ash-soaked lava, and that the enclosed blocks of Sara-isi are fragments of pre-existing rocks or lapilli.

8. And a portion of the volcanic vent was disturbed and shattered by explosive action; and disrupted rock fragments thus formed as well as the surfacial volcanic fragmental materials became coated with newly erupted lava mingled with ashes; they were then thrown out and dashed against the surface of the ash-field upon falling. The lava coating on each block was then still in a semifluid state, so that the upward-projecting rim of the crust was produced by the impact action. . . . . .

9. But the heat effects on the core rock-fragments are slight. There is, however, no sign of fusion of the enclosure. . . . . .

Prof. T. Matsumoto,[1] too, seems to adopt No. 7 among the above illustrations. Prof. Harada proceeds further[2]: we can in some measure imagine the volcanic activity of that time by means of the extreme fluidity of the crust lava which perhaps became temporally very great due to some actions during the crystallisation of magma, and by this occasion Sara-isi was then probably produced *extremely accidentally*. . . . . .

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1. 松本唯一:—大阿蘇の新研究 (1932) p. 36.
2. 原田準平:—或種の火山拋出物と火山活動に就て, 地質學雜誌 **34**, 208 (1927).

Undoubtedly it can be said that he made a step of progress in the research of Sara-isi, yet his illustrations about the method of formation are not much different from those of Prof. Sato and Igi. One assumes the whole body of Sara-isi being in a plastic state just before falling on the field, and the other assumes the crust only being in a plastic state while its core being the pre-existing rock material. Both adopt in the same manner that a piece of semi-plastic magma falls on the ash field and its margin will be lifted mechanically due to its impact with the ground.

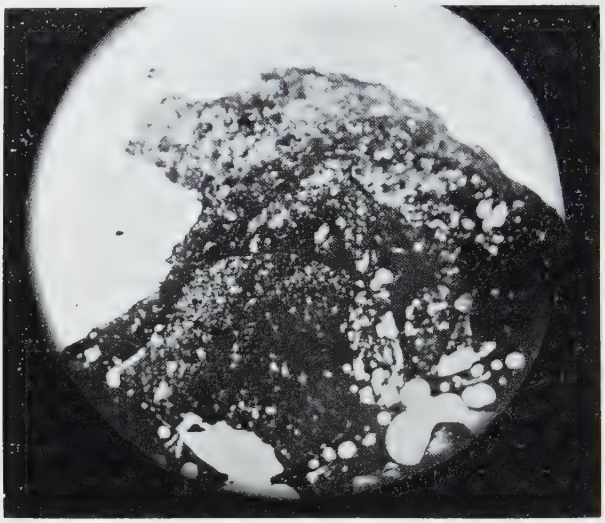
The present writer, however, prospecting the places and manners of existence of Sara-isi during several years, and investigating carefully their structures, found out many facts contradictory to the above illustrations, and wanted a new theory of the origin of Sara-isi. For these reasons we will at first revise and enlarge the investigations on the nature of the Sara-isi itself, and then propose our opinion about the method of its formation based upon many decisive facts newly found. Lastly we will discuss the peculiarity of the volcanic activity of Aso, suggested by the Sara-isi production which is limited to that place. Indeed according to the historical records and our observations during several years it is undoubtedly a fact that, at Naka-dake crater alternative explosions and accumulations of hot water pools are constantly repeated; in other words, it is a notable feature of the action at Aso volcano that both states of an active volcano and a crater lake are often repeated alternately.

Thus we propose to take the mode of volcanic activity with the above feature as a new one named, "the activity of Aso-type." Further, applying our new view of Sara-isi on Naka-dake, Kisima-dake and Ô-jo-dake, we will make an atempt to illustrate the respective peculiarities and the orders of action of those craters.

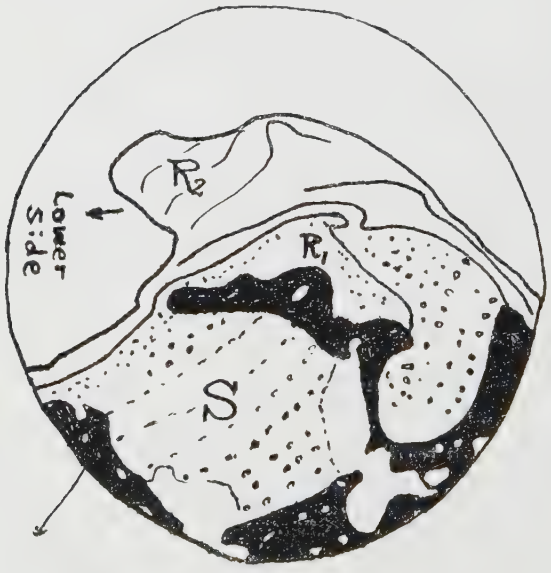
§ 2. Yield Manners and Discrimination of upper and lower Sides of Sara-isi

The sides, towards which the rim of Sara-isi bends, is hitherto called "its upper side" and the other "its lower side." This expression seems to me a view beguiled due to the word "Sara" (dish). As for the scientific research, indeed, the expression must be discriminated carefully by the actual observations of its normal state of existence, because this discrimination may support not a little the view of the formation of Sara-isi. If one carefully observes the yield

Fig. 10. Section of a SARA-ISI to show the growth of a secondary rim toward the earth.



S : — volcanic ash arranged horizontally.
R_1 : — primary rim.
R_2 : — secondary rim.



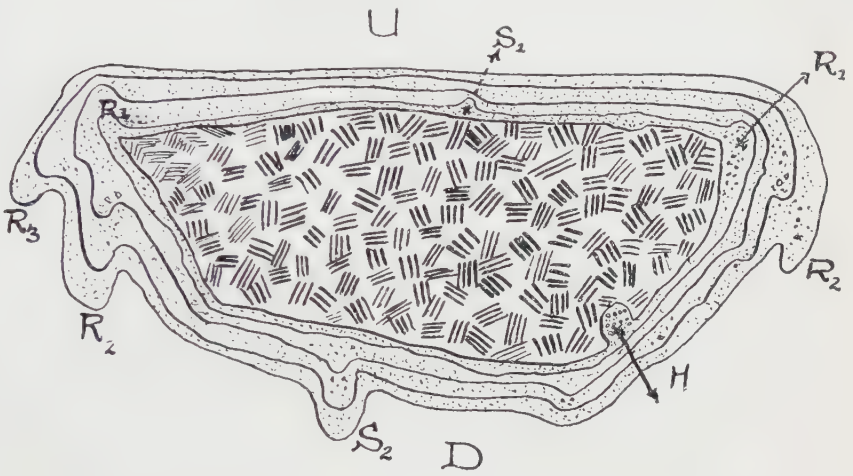
Core material

manner of Sara-isi at the field, he will find that the rim generally suspends down toward the ground except in rare cases in which the original state seems to have been disturbed, and that in those exceptional cases another new rim is growing toward the ground at the margin (Fig. 10).

Sara-isi with a well developed rim can be found more commonly on rather pebbly slopes than on sandy fields, and those lying on geographically old fields have developed better than those on younger fields. By these facts of observation, it seems to me more reasonable to consider that the direction of the main rim should be called "the upper-side (ridge) direction" (Fig. 11). Careless discrimination of the sides, perhaps

obliged many of the seniors to attribute the formation of Sara-isi to the falling impact of a pseudo-plastic magma against the ground.

Fig. 11. Schematic representation of a SARA-ISI.



U : — upper side.
R_2 : — main rim.
H : — layer of Yona in a hole.
core material.
tufaceous part.
D : — lower side.
S_2 : — stalactitic process.
R_1 : — primary coagulated part.
coagulated part.
flow band.

§ 3. The Out Side of SARA-ISI

The upper-side of a Sara-isi is dark gray or dark brown and rather smooth, and its protuberances if they exist are dull. The part which is covered by soil or rock fragments, is commonly yellowish brown like the colour of the lower side.

The lower-side, on the contrary, is commonly yellowish brown or white gray and is ragged. The portion which does not directly contact with the ground is comparatively compact and has a colour near that of the upper side, and it must be noticed that there are quite a few stalactitic processes on such a portion. A small Sara-isi (1–2 cm. in diameter) is sometimes smooth on its whole surface, and its rim is not developed enough to discriminate the sides, though even in such a difficult case we will not fail to discriminate, if we carefully examine its colour. There may of course be a complicated Sara-isi whose sides can be discriminated only by a trained man.

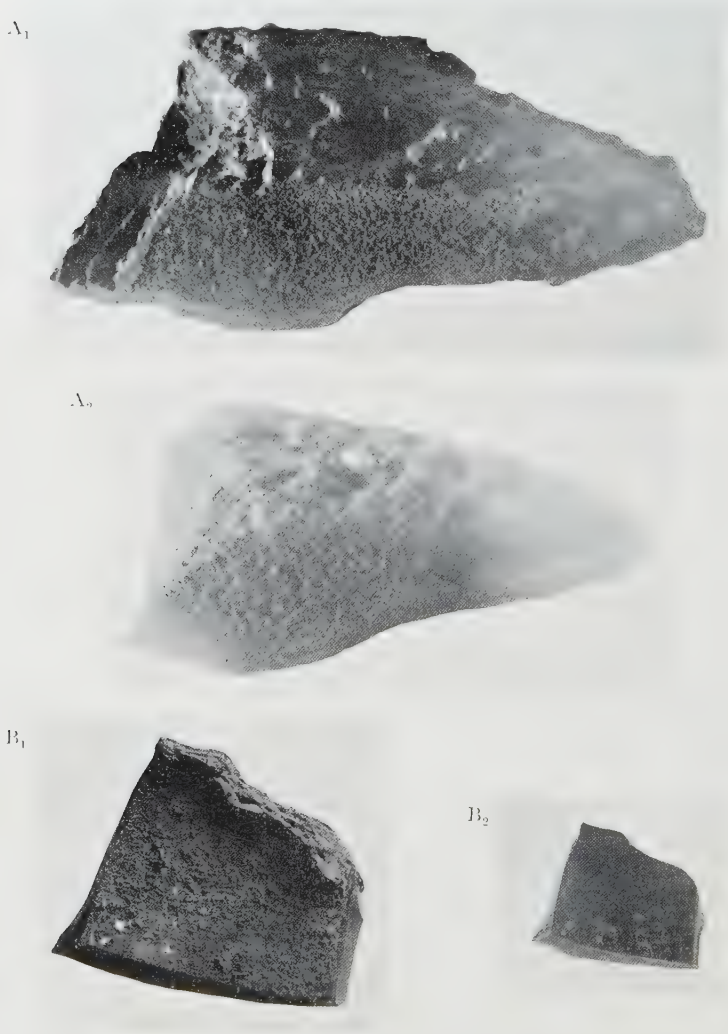
Fig. 1, 2, 3, 4, 5, 6 show both sides of Sara-isi.

Fig. 4 shows the stalactitic processes on the lower side.

§ 4. Sara-isi is Composed of Two Parts

Divide a Sara-isi into two pieces. Observing a part directly by the naked eye or under the microscope, it can easily be recognized that the construction of the core is entirely different from those of the crust and rim (Fig. 5, 6). In the next place when we treat the other part for several hours or more with hot concentrated chloric,

Fig. 12. SARA-ISI under the chemical treatment.



A_1, B_1 : — original shape.
A_2, B_2 : — after the treatment by NaOH solution.

sulphuric, nitric acid, or even with hot aqua regia, we can not see any mechanical variation as a whole. But if we put a Sara-isi into a hot caustic soda solution (1–5 Normal), there occurs at once the destruction of the rim and crust, but the action will cease in about ten or twenty minutes. Then wash the adhering $Fe(OH)_2$ off the residual solid block with HCl and next with water. Now investigate this solid block macroscopically or microscopically, then we will find surprisingly that the core part in the former piece was unchanged while the crust completely disappeared.

For the sake of convenience, therefore, we will call the residual part "the core of the Sara-isi" and the other collapsed part "the crust or coagulated part," and then a Sara-isi may be discriminated roughly into two main parts — the crust and the core.

Moreover from the above experiment we see that the rim and the stalactitic prominences being undoubtedly to the crust part (Fig. 12).

§ 5. The Crust or Coagulated Part of a SARA-ISI

The crust is the most important part for the investigation of the origin of Sara-isi. It may rather be said that the genesis of Sara-isi can be revealed only by the investigation of the construction of the crust. Following experiments were carried on with such a view in mind.

(1) If a Sara-isi whose enclosure is obviously a vitreous lapillii (Fig. 2) is heated up to about 1100°C in an electric furnace, its crust becomes reddish due to Fe_2O_3, chapped, and somewhat brittle, but original shape is maintained, while the core begins to melt. With such Sara-isi, therefore, it is difficult to conjecture as Prof. Harada did that the crust is composed of a highly fluidal magma. The temperature at which the original shape begins to go out was kindly measured for me by Dr. Akiyoshi Tadokoro, Engineer of Yawata Seitetsu-sho, as :—

crust . . . . . 1175–1190°C.
core . . . . . 1165°C.

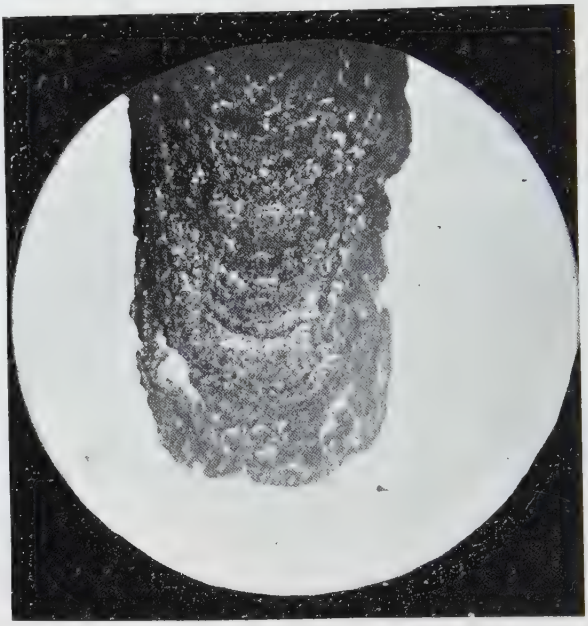
(2) From our witnessing the falling habits of bombs in the cases of recent eruptions of the Aso-volcano and some laboratory experiments we are led to conclude that when a highly plastic matter is dashed down on an ash field, its upper side (the surface which is in contact directly with the atmosphere) is generally more smooth than

its lower side (the surface which is in contact with the ground). Moreover stalactitic prominences are not produced in the upper smooth flat side. These facts will prove that the upper and lower sides of a Sara-isi are scientifically reverse to those of former investigations.

(3) The vertical section of a Sara-isi shows under the microscope some alternative stratum — a rough tufaceous layer and a compact brown fluxion layer (Fig. 5, 6, 10).

One Sara-isi from Sara-yama possesses a crust composed of about twenty layers. Prof. Harada does not point out such alternative stratum. Therefore his illustrations of the method of formation of the crust (due to coating of magma mingled with some Yona) look reasonable, but can not stand in our case. We commonly find

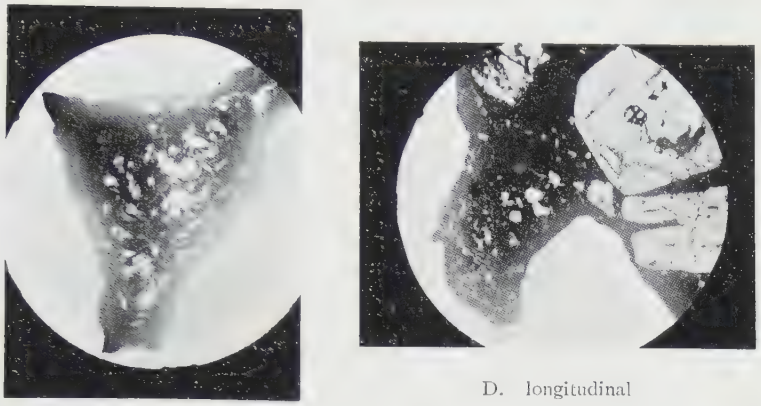
Fig. 13. Section of the lava stalactitic prominences.



A. longitudinal

B. cross-section

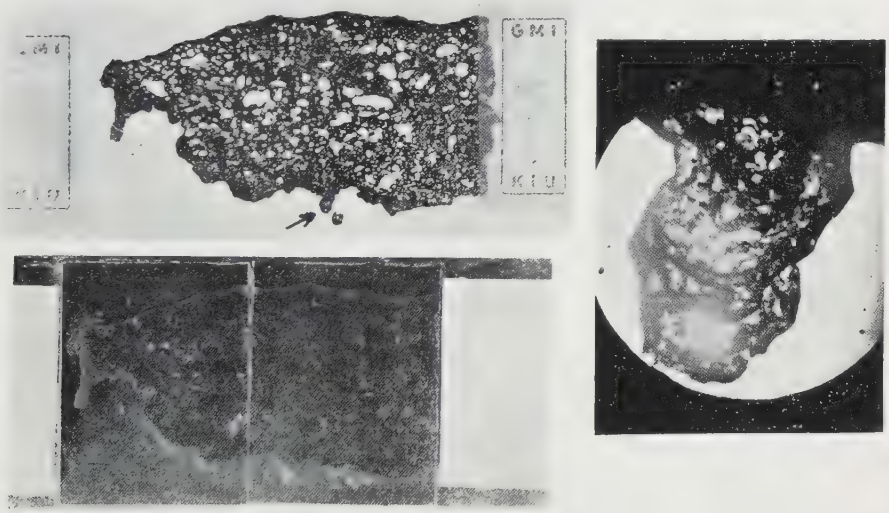
Fig. 13



C. longitudinal

D. longitudinal
(Narao-dake)

Fig. 14. Section of SARA-ISI and its lava stalactitic process.

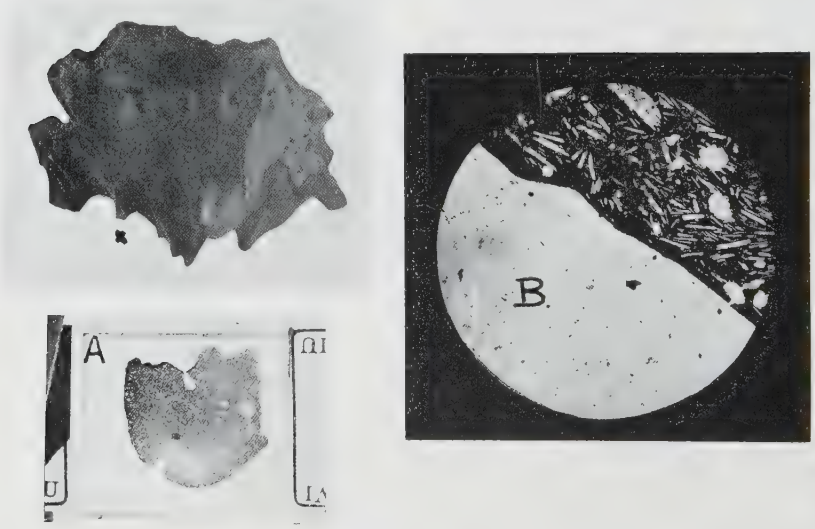


some differences between the tufaceous layers, but not among the fluxion layers. The tufaceous layer is composed of fragmental pieces of volcanic ash materials (about 1/10 m.m. in diameter) cemmented with microparticles (less than 1/1000 m.m. ca.) and Fe_2O_3. The volcanic ash is mainly composed of the hitherto known anorthite, augite, volcanic glass, rock, olivine, magnetite etc. When we remove ferric oxides by chloric acid from the section, the brown fluxion layer becomes white (colourless) under a low power microscope. If they

were brown glass, such a change could not occur. Under a high power microscope (greater than 800 times ca.) we see crowded amorphous microparticles which we conclude to be quartz particles. Now if we trace the layer under a low power microscope, they form closed curves as shown in the figure schematically (Fig. 12).

Each layer does not disturb its order, though the width is of course not uniform. This relation holds also in the case of poly-rimmed Sara-isi or of the stalactitic prominences. The cross-section of a stalactitic prominence shows some concentric circular structures as those of a stalactite (Fig. 13, 14); on the contrary, a protuberance of the core does not show such a structure (Fig. 13 D). Similar

Fig. 15. Lava stalactite from Mt. Fuji.



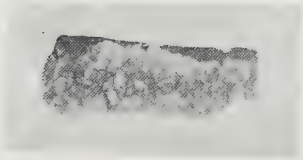
A : — the cross section.
B : — under the microscope.

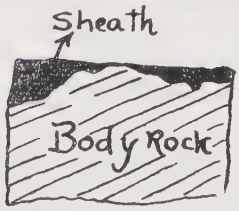
structures can not be found also in the section of a lava stalactite from Mt. Fugi (Fig. 15). These facts also play an important role in the investigation of Sara-isi genesis.

(4) The method of formation of the crust part may inductively be said to be different from that of the core part as already stated above. Now let a rim represent the crust. It can be easily crumbled by NaOH (5N) solution in about twenty minutes; and there remains a residue of chiefly volcanic ash (Yona) and $Fe(OH)_3$, while the filtrate contains SiO_2 (principal part), a few almina and traces of Ca, Na, K

and Mg. But by any acid, HNO_3 (conc.) or aqua regia — cold or hot —, HCl (conc.), H_2SO_4 (conc.), the construction of the rim can not be broken down though the rim is only 0.1 m.m. thick; it only becomes white, Fe_2O_3 being removed. This will show that ferric oxide, calcium carbonate, or sulphate are not the principal cementing materials, and that the fluxion layer may originally be white (colourless)

Fig. 16. Tume-isi or Lava-sheath from the New-Crater (1906).





Section of a sheath

but becomes later brown due mainly to Fe_2O_3. The remarkable crumbling phenomena, as with the rim of Sara-isi, does not occur with any products of plastic magma, e. g., Aso-lava, volcanic bombs from the great Aso ancient somma, bombs from Taka-dake, Kisima-dake, Nara-o-dake, old bombs from Naka-dake, new bombs from the fourth crater of Naka-dake, and "Tume-isi" or Lava-sheath (a curious stone from the New-Crater of 1906) (Fig. 16). At my request, various analyses of volcanic products were kindly performed by Mr. T. Kido, formerly chemist of our laboratory, to whom the writer expresses his best thanks here.

Table 1. Chemical Composition of the Rim of Sara-isi (sample from Fig. 2), by T. Kido.

	Soluble part in NaOH(5N-) 5 hours 0.442 gr.	Residue 0.558 gr.	Total 1.000 gr.
SiO_2	37.080%	26.860%	63.940%
Al_2O_3	2.320	8.860	11.180
Fe_2O_3	—	11.660	11.660
CaO	0.800	4.500	5.300
MgO	0.420	—	0.420
SO_4	5.753	0.106	5.859
—	—	—	—
$\frac{SiO_2}{Al_2O_3}$	16.1	3.2	6.2

Table 2. A bomb (Nov. 1929) from the fourth-crater, ASO.

SiO_2	53.8782%
Al_2O_3	16.4302
Fe_2O_3	5.6907
CaO	3.5966
MgO	0.7581
SO_4	0.4142
—	—
$\frac{SiO_2}{Al_2O_3}$	3.3

Table 3. Volcanic ash (1914) from Sakura-jima.

SiO_2	61.4120%
Al_2O_3	18.4321
Fe_2O_3	6.8179
CaO	3.9251
MgO	1.0033
SO_4	0.5342
—	—
$\frac{SiO_2}{Al_2O_3}$	3.3

Table 4. YONA from ASO.

	27–28 Aug. 1929 from 4th crater, lain on our laboratory.	5–8 Nov. 1929 from 4th crater, lain on its rim.	4–8 Sept. 1930 from 1st crater, lain on its rim.	7th Sept. 1932 from 1st crater, lain on its rim.	10th Dec. 1932 from 1st crater, lain on Tochi-no-ki village.
SiO_2	52.28%	57.53%	51.21%	51.77%	54.34%
Al_2O_3	8.23	12.44	8.48	5.92	6.81
Fe_2O_3	9.47	8.74	6.88	11.45	11.13
CaO	6.30	5.56	5.24	5.53	6.02
MgO	0.66	0.65	1.19	1.69	1.54
SO_4	0.33	3.22	0.74	0.95	2.40
—	—	—	—	—	—
$\frac{SiO_2}{Al_2O_3}$	6.9	4.6	6.0	8.8	8.0

Table 5. Glass-cored Sara-isi (Fig. 17).

Wt. of the sample . . . . . 3.3432 gr.
Wt. of the core (glass) . . . . . 3.2934 gr.
Then coagulated part . . . . . 0.0498 gr.

	Soluble part in KOH(5N), 5 hours	Residue
SiO_2	54.4177%	24.8996%
Al_2O_3	0	4.4724
Fe_2O_3	0	11.2353
CaO	0	6.3426
MgO	0	0.9209
SO_4	2.8094	0.3305
—	—	—
$\frac{SiO_2}{Al_2O_3}$	great	5.6

Table 6. Scoriaceous Lapilli (1928) from the Fourth Crater, ASO, by Tsuya.[1]

SiO_2	53.53 wt. %
Al_2O_3	18.67
Fe_2O_3	2.67
FeO	5.93
CaO	9.27
MgO	3.80
—	—
—	—
$\frac{SiO_2}{Al_2O_3}$	3

Table 7. Volcanic Ash (1914) from the volcano Sakurajima accumulated on the play-ground, Kagoshima Normal School.[2]

	By the Geological Survey of Japan	By Eng. Depart. Kyusyu I. Univ.
SiO_2	63.39%	60.38%
Al_2O_3	16.75	18.10
Fe_2O_3	3.100	8.49
FeO	4.00	—
CaO	5.38	6.01
MgO	1.43	3.61
—	—	—
—	—	—
$\frac{SiO_2}{Al_2O_3}$	3.8	3.3

Table 8. The Average Chemical Composition of Igneous Rocks.[3]

	Earth	Japan
SiO_2	59.12	61.92
Al_2O_3	15.34	15.72
Fe_2O_3	3.08	2.07
FeO	3.80	4.41
CaO	5.08	5.16
MgO	3.49	2.47
—	—	—
—	—	—
$\frac{SiO_2}{Al_2O_3}$	4	4

Table 9. Chief Activities of the ASO-crater.

1928. Sept. 6 . . . . . during about one week (the 4th. crater).
Dec. 19 . . . . . „
1929. Jan. 22 . . . . . „
Feb. 20 . . . . . „

1. Tsuya:—On the recent ejecta of Volcano ASO, 地質學雜誌, **36**, 17 (1929).
2. 地質學雜誌 **21**, 540 (1914).
3. H. S. Washington a. F. W. Clarke:—Proc. Nat. Acad. Sci. **8**, 108 (1922).

Table 9. (Continued)

1929. Mar.	6	during about one week (the 4th. crater)	
	18	„	
	29	„	
May	9	„	
June	22	„	
Ju'y	11	„	parasitic crater appeared.
	25	„	
Aug.	24	„	
Sept.	14	„	
Oct.	6	„	
	22	„	
Nov.	4	„	after this activity crater pool occurred.
1930. Sept.	4	„	after this time the crater pool reappeared in the *4th* crater, and its eruption *ceased.*
1932. Sept.	4	the *1st* crater awakened again after its *long repose* (about 30 years).	
Oct.	10		
Dec.	17		
1933. Feb.	24—Mar. 8, *greatest* explosion during recent years		
May	5		
Sept.	4		
1934. Jan.	3	now *1st* crater became to be a pool.	

For the bombs, Mr. Kido's results of analysis are sufficiently agreeable with those of Tsuya for scoriaceous lapillii (Table 2, 6), and also with those of the Geological Survey of Japan and the Engineering Department of the Kyusyu Imp. University for the volcanic ash from Sakura-jima (Table 3, 7). All these results give the ratio of SiO_2 to Al_2O_3 as about 3–4, which is coincident with the average for the igneous rocks of all Japan or the earth (Table 8). For the rim of a Sara-isi, however, the ratio of SiO_2 to Al_2O_3 is about 6 as a whole, while in the solute with NaOH (5N) about 16, and in its residue about 3 as those for lapillii (Table 1). This fact seems to me to show that *the rim of a Sara-isi is not a block magma, but a cluster of Yona (volcanic ash).*

The reader will agree with us on this point, if he looks at the analysis of Yona in Table 4. Especially the ratio in Table 5 is extraordinarily great.

Now, we can see that SiO_2 is found more abundantly in the Yona erupted at the first stage of an eruption than at the last stage. For example, in the Yona of 27–28 Aug. 1929 the ratio of SiO_2 to Al_2O_3 is 6.9 while in that of 4–8 Nov. 1929 is 4.6, which is very near to those of lapillii. Again, in the Yona of 4–8 Sept. 1930 (after about ten months repose) it increases to 6.0. For the sample of the *1st*

crater also, the above relations hold obviously; for example, in the Yona on 7 Sept. 1932 (first stage after a long repose of about 30 years) the ratio is 8.8 while in that on 10 Dec. 1932 the ratio decreases to 8.0. The volcanic ash from Sakura-jima may perhaps be newly generated directly from the magma, because its ratio of SiO_2 to Al_2O_3 is very near to that of a lapillii.

Fig. 17. SARA-ISI which enclosed a glass piece.

lower side.

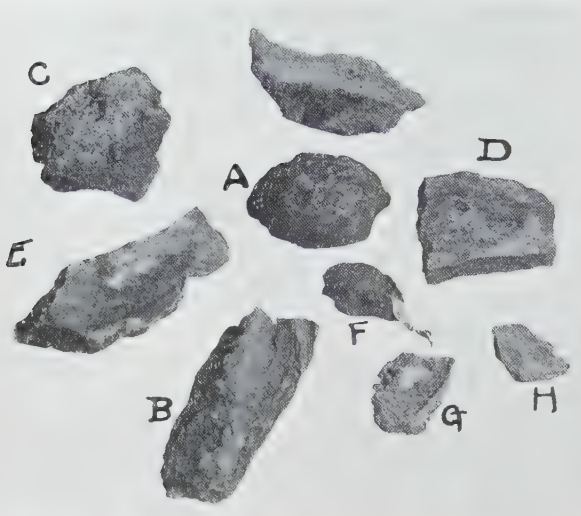
under the microscope.

§ 6. The Core of a SARA-ISI

The core of a Sara-isi is commonly a volcanic ejectamenta as Prof. Harada has already reported (Fig. 1, 2, 3, 4, 5, 6, 7, 8, 10, 18), that is, it may be a magma fragment, a bomb, a lapillii, a tuff or pre-existing rock fragment. For all our efforts we failed to discover a lapillii of "ash mingled magma" which was to be expected from Harada's view. According to our view that the rim is of posterior nature, we may expect the existence of Sara-isi with a core of some artificial substance; and the writer made his utmost efforts to find such a Sara-isi and at last successful results have been obtained.

If we prospect carefully along the flank of the

Fig. 18. Some kinds of SARA-ISI.



A: its core is a spindle bomb.
B: „ a twisted bomb.
C: „ a massive rock, on whose lower side many lava stalactitic prominences grow.
D: flat one.
E: irregular one.
F: symmetrical one.
G, H: other irregular ones.

ancient crater of Naka-dake we will surprisingly discover many suspensions at the margins of grave stones, stone buddhas, stone monuments, stone lanterns, stone wash basins, etc. (Fig. 19, 20).

These suspensions have a quite similar form and composition as that of a Sara-isi. More decisively we have found a number of Sara-isi whose core are broken pieces of glass bottles or earthen ware. (Fig. 17, 21). Fig. 17, 11B shows examples whose rims are developed along the margin of pieces of a beer bottle. They were collected at the west flank of Naka-dake ancient crater by the writer on 15th Nov. 1931.

Thus, the cores of Sara-isi are not only the ejecta from the crater but the rock fragments from the crater flank as already reported, and more surprisingly some artificial substances also.

§ 7. The Formation-process of the Rim of a SARA-ISI

Many facts described above will entirely refute the customary explanation of the genesis of Sara-isi, and therefore we are obliged to construct a new illustration for the formation-process, basing it upon the facts discovered recently. The writer believes that "the rim of Sara-isi" is not produced directly by the effects of a volcanic outburst, but it is rather a secondary posterior production. Fine pieces

of volcanic ash (Yona) are coagulated by colloidal silicic acid on some of the rock fragments of the ground and hardened to form the rim and crust of a Sara-isi. If the writer's view is true, then it must follow :—

Fig. 19. Stone lantern erected on 1637, and the section of its margin.

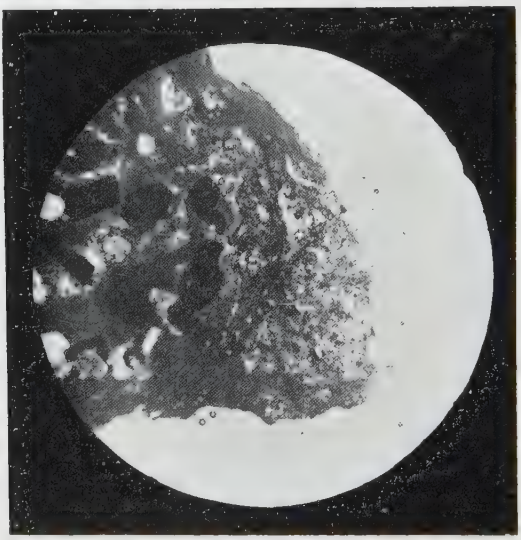
i) The core of Sara-isi may not only be such ejectamenta as a volcanic bomb, a pre-existing rock fragment from the crater, or a tufaceous matter, but may also be some foreign fragment carried by nature or a human being. Indeed the same coagulating matter as that of the rim of a Sara-isi will grow on some materials, on whose surface colloidal silicic acid can coagulate.

ii) The crust of Sara-isi will grow with time due to the piling up of colloidal silicic acid. The rim should be usually suspended downwards.

iii) Sara-isi will be produced at the place where colloidal silicic acid and pieces of volcanic ash are supplied, and on condition of the hardening of the colloidal silicic acid by the leaving of its water. Now if we can show that these three circumstances really exist, our view for the genesis of Sara-isi will be readily accepted by the readers, so let us proceed to explain these points by the actual facts taken from the volcano Aso.

I). *The Formation of the rim and crust—Coagulating layer—is not restricted only on the volcanic ejecta.*

Fig. 20. Stone bowl curved on 1807, and the section of its margin.

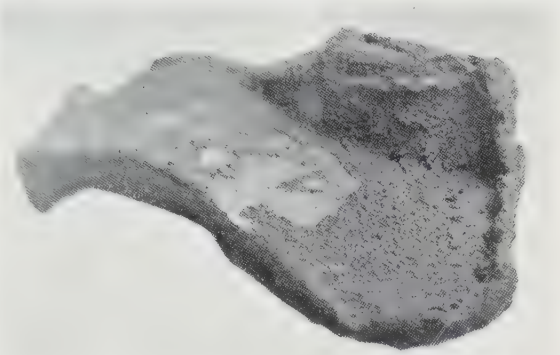


This is verified by facts in § 6. We add here another interesting fact that numerous stalactitic processes are actually developing in some caves under a lava flow and their physical and chemical natures are entirely similar to those of the rim or stalactitic process of a Sara-isi (Fig. 22).

II) *The crust of a Sara-isi may grow with time.*

This will be indicated by the stratification of the rim or crust of a Sara-isi. We will here again add a few new facts. If we examine carefully, we will find that the Sara-isi found at Sara-yama are most developed, those from Sensui-yama come next, and those from the west flank are the most incomplete.

Fig. 21. SARA-ISI which enclosed a stone ware piece.



On the other hand, considered geologically, Sara-yama is the most ancient while the west flank is the newest. Hence the above will show that Sara-isi may grow with time.

Moreover the rims developed at the stone lantern erected in 1637 and at the stone bowl carved in 1807 indicate clear stratifications under a microscope, and we can estimate the rate of growth from the age of their erection. Estimation gives the rate of two fluxional and two tufaceous layers in about 300 years . . . . of course very roughly.

The fluxional layer may be coagulated in a comparatively calm period of the volcano, while the tufaceous layer seems to develope

Fig. 22. Numerous posterior stalactitic (and stalagmitic) processes under a lava flow.

in the active period, accumulating volcanic ash pieces mingled with colloidal silicic acid; thus the above rate might suggest that the activity of Naka-dake prospered and decayed in about 150–160 years (very roughly). This number approximately coincides with that obtained by the statistics of activity in the historical records,[1] and the present time seems to be a comparatively active period.

III) *Production of colloidal silicic acid is possible in the ASO-crater.*

Production of colloidal silicic acid is possible in a crater where hot water stays in its bottom through which sulphuretted gas effuses violently and sulphur or sulphuric acid result. Because it is a well known fact that sulphuric acid becomes neutralized decomposing the volcanic rock or ashes, and colloidal silicic acid can be well produced when the concentration of H ions is suitably decreased. If ashes (Yona) mingled with colloidal silicic acid are blown out from the crater and fall on the rocky ground, then a film is resulted due to the colloidal substances. Moreover, the residual sulphur or sulphuric acid, which were spread out with the above materials, will stick to the rocks and may decompose the rock materials in many years in the same manner as in a hot crater pool. Then if the acid is suitably neutralized, the colloidal silicic acid becomes fixed to the rocks, losing the water in the sun. The stratiform crust may occur only by the pulsating supply of the colloidal silicic acid, i. e. only when both the effusion of volcanic ash and the accumulation of hot water in the crater are repeated alternately. The growth of the crust will depend on the frequency of the repetition of the alternate states of effusion and hot water accumulation in the crater. We here emphasize that these conditions are satisfied by Aso both in the history and also at present as seen in the next chapter.

§ 8. The Characteristics of the ASO-type Activity

If Sara-isi is a special product from volcano Aso, it may inversely be said that the activity of volcano Aso has a certain singularity. And if we find Sara-isi in some other volcano, we may say that the volcano makes activity like that of Aso. We shall now summarize the characteristics of volcano Aso. "Eruptions occur very frequently. During the repose of eruption, colloidal silicic acid is produced in the crater where hot water stays in the basin and sulphuretted gas

1. 震災豫防調査會報告 (日本噴火誌下) 第87號 p. 28.

effuses through the pool violently. The eruption is rather moderate. If the eruption is very severe, the growth of the crust of Sara-isi must be disturbed by the abundant ejecta . . . Sara-isi must be buried under the abundant Yona. We will call this type of activity shortly "*a volcanic activity of Aso-type.*" The above characteristics were well experienced by the writer during the several years since he began his study on Aso. The volcanic activities of Aso recorded in Japanese history also may be said to show completely the same characteristics. The oldest Aso-activity in the record is written as 延暦十五年秋七月辛亥詔曰比來大宰府言肥後國阿蘇山土有池其名曰神靈池. 水量經年未嘗增減而今無故涸減二十餘丈云々. (日本後紀).

Since this time, there are numerous records which show that this volcano repeated the bursting out of Yona (霾, volcanic ash) in the active period and the pool-state (神靈池) of Kusui (苦水, active hot water) of the crater in the calm period.

Indeed the repetition of "eruption" and "the presence of hot water" in the crater was surprisingly numerous, and the extraordinary adhesive power of Kusui (苦水) was known already since very long and recorded as "苦水沸騰空中東西洒落其東方者如布延縵廣十許町水色如漿黏著草木雖旬日不消解 (肥後國誌).

Moreover, we can also suppose the existence of the above two phases of action from the etymological meaning of the name of the mountain . . . Aso.

Dr. Tanimoto[1] was said to have solved the word *Aso* as coming from the Ainu language with the meaning "mountain blowing out the fire" as ASAMA-yama. Dr. John Batchelor says that Aso means "a crater"[2] but in other place he says that Aso-yama . . . . 阿蘇山 . . . . Aso-nupuri means "opposite mountain."[3] Dr. T. Oda[4] says that Aso may be an abbreviated word from 阿蘇羅 (violent). For the present Naka-dake also, various different names can be found in the ancient literature, such as:

ASO . . . . . (阿　蘇 . . . 日本書紀, 阿曾 . . 和名抄, 嶽宗 . . 筑紫風土記)
O-take or *Mi*-take . . . (御　嶽 . . . 肥後國誌)
Mae-dake . . . . . (前　嶽 . . . 阿蘇郡誌)

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1. 阿蘇郡誌 (1926) p. 558.
2. Dr. John Batchelor:—Key to the Study of Ancient Japanese Place Names (1929).
3. „ „ :—The Fit-Dwellers of Hokkaido and Ainu Place Names considered (1925).
4. 織田得能:—佛教大辭典.

Nara-O-dake . . . . . (楢尾嶽 . . . 肥後國誌, 阿蘇郡誌)
Nara-yama . . . . . (楢 山 . . . 同 上)

These may be grouped into two words :—" ASO-O-MAI " and " NARA." " ASO-O-MAI " means " a place where a firing crater exists " and " NARA " means " a ditch " in the Ainu language. Thus we imagine that the two phases of action (eruption and water-staying)

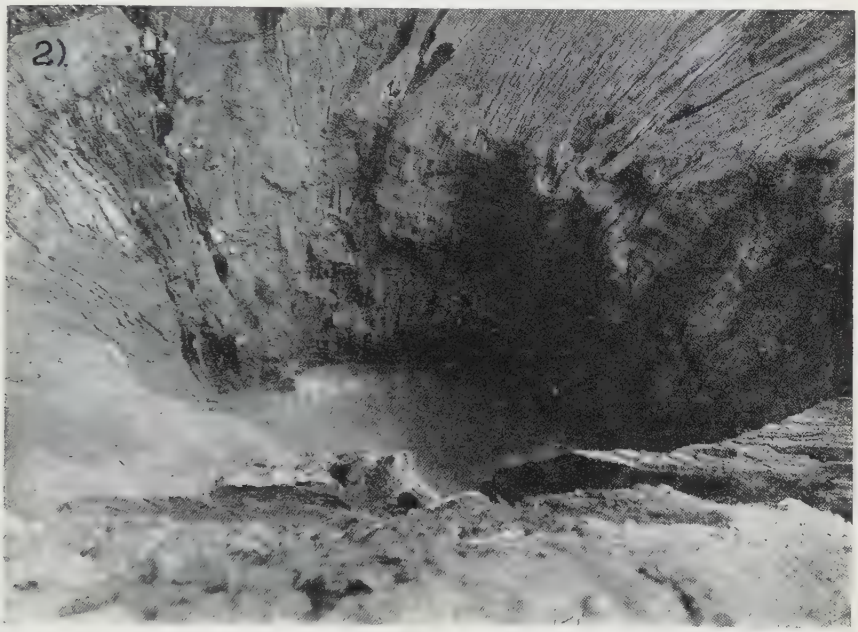
Fig. 23. Photograph of the two phases of action in the ASO-Craters.
1) Photo. on *4th* Nov. 1929.

I & II-Craters :—in state of hot water pool with solfataric action. IIII-Crater :—in explosion.

repeated since prehistorical time so frequently that the ancient people noticed the different phases and named the mountain in two different meaning naturally. If that is the case, the writer can expect some other words of a similar kind which we will speak of here. YONA (霾) (a peculiar word in the district of Aso) seems to be altered from UINA meaning " the volcanic ash," and KU-SUI (苦水) also a word altered from " KUSURI " meaning " the hot spring " in the Ainu language. Neko-dake is the famous cone among the so-called ASO-GO-GAKU (five famous central cones in the great Aso), and has a splendid shape like the teeth of a saw. One of its old names is " IHREME-USI " (肥後國誌). This word seems to me to be

altered from "PIRI-EMUSI," which means "a sword shape mountain whose edge is nicked." At the volcanoes Kiri-sima,[1] Asama[2] and others their craters are commonly called "O-KAMA" (the kettle) or "O-HACHI" (the pot) while at the volcano Aso they are called "O-IKE" (the pool). This may perhaps be due to such reasons that the craters of the other volcanoes except Aso were dry and the water

Fig. 23. Photograph of the two phases of action in the Aso-Craters.
2) Photo. on *6th* Oct. 1931.



IIII-Crater:—becoming a hot water pool this time.

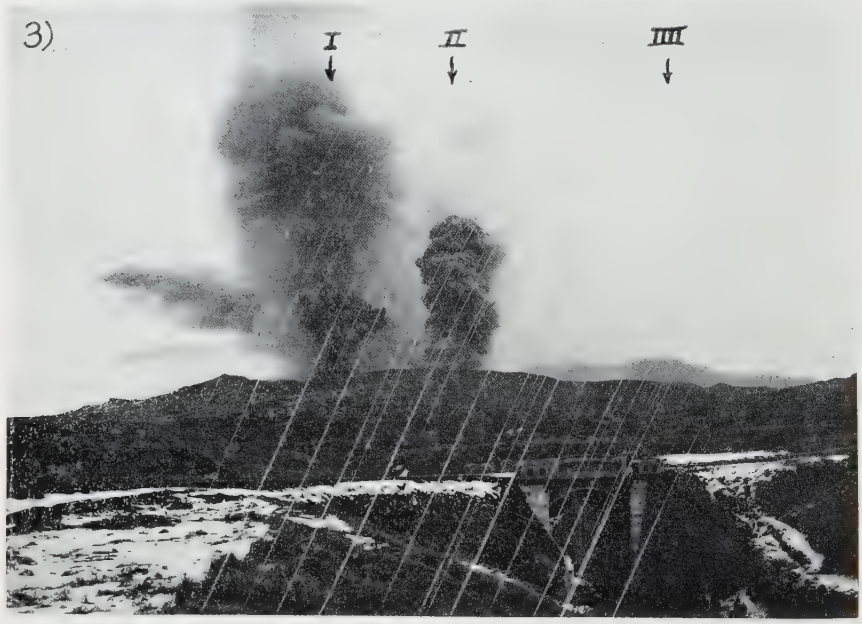
was scarcely staying in the basin even during the repose-period.

From the above reasoning, the readers may understand that the characteristic activity of Aso has been continued since far remote times, so that Sara-isi may be produced inaccidentally.

We tabulated the chief activities since 1928 in Table 9, and the two phases of activity of the craters *4th* and *1st* are photographed in Fig. 23 (1, 2, 3).

1. This name seems to me to be altered from "Ye-Keure-So-Oma" and "Karuru-Keure-Guru" which mean "a mountain from whose crater the pumices and ashes burst out."
2. This seems to me to be altered from "ASO-Oma" which means "a mountain which has a firing crater." Dr. J. Matsumura considered it to mean "a sparkling crater" in his "An Etymological Vocabulary of the Yamato Language (1921)."

Fig. 23. Photograph of the two phases of action in the Aso-Craters.
3) Photo. on 9h. *2nd.* Mar. 1932.



I & II-Craters:—in explosion. IIII-Crater:—in a hot-water-pool state.

§ 9. Volcano Kisima-dake also had a Period of ASO-type-activity.

From the preceding discussions, we may suppose that a volcanic crater has once at least "activity of Aso-type" if we can find Sara-isi in its surroundings. Now, in the 1/50000 map of Aso issued by our Military Land Survey, the west cone of the ASO-GO-GAKU is named Kisima-dake (Fig. 9). If we search round the circumference of this cone, we can collect Sara-isi at the top of the volcano Ô-jo-dake which is on the NE side of Kisima-dake (Fig. 9) and at a part of the somma of the explosion crater Senri-hama (Fig. 9). The relation of these newly found habitats to Kisima-dake corresponds to that of Sara-yama and others to the Naka-dake craters.

Thus, volcano Kisima-dake must have once had Aso-type activity, and moreover it will be younger than Ô-jo-dake or the Somma of the Senri-hama explosion crater.

This conclusion may be supported etymologically also. According to 肥後國誌, Kishima-dake has many old names such as "Donben-

dake," "Doben-dake," "Tama-dake," "Suisho-dake," etc. These seem te me to be altered from three words with different meanings. One is "To-Un-Pe" meaning "a mountain which has a pool," and next "To-Oma" meaning "a mountain at which a pool exist," and the last "Shui-sho" meaning "a mountain which has a crater." Namely, this cone has two kinds of names as those of Volcano Naka-dake, indicating two phases of activity peculiar to the Aso-type-activity. As far as we know, there is no literature to determine geologically the order of volcanic repose of the two cones, Kisima-dake and Ô-jo-dake; but fortunately our studies on Sara-isi throw light on this point.

In conclusion the writer wishes to express his sincere thanks to Prof. Dr. T. Nomitsu for his kind instruction and advice throughout the work.

A Method of measuring the Net Density of a Powdery Substance Which dissolves a Little in a Liquid

By Usaburo Yoshida, Sin-ichi Matano and Masaru Watanabe

(Received March 22, 1936)

Abstract

A method of measuring the net density of a powdery substance by immersing in a liquid which dissolves a small quantity of the substance, is described. The net density of picric acid is measured by this method by immersing it in water, and the values of 1.7642 was obtained for purified picric acid.

Some years ago Mr. Takei and one of the writers[1] devised a method of measuring the net density of a fibrous, powdery or porous substance which is liable to occlude gases and water. The essential part of that method is to evacuate off all the occluded gases and water from the specimen. With this method we must, of course, select a proper liquid which does not dissolve the solid specimen as is usually the case. But it occurs sometimes, with some specimens, that it is difficult to select such a liquid which does not dissolve the specimen entirely and that we are obliged to use a liquid into which dissolves a small part of the solid specimen. The present method consists of giving a proper correction in such occasion.

As the method is essentially the same as before an explanation of it will be repeated briefly in the following for the sake of convenience. Several grams of the specimen are put into a glass tube which is sealed at one end as is shown by A in Fig. 1. Then the tube is necked with a burner several centimetres from its sealed end as is shown by B in the same figure. Sufficient care is to be taken, in this case, not to heat the specimen contained in the sealed end. The tube is scraped with a file at *a* and *b* in B, Fig. 1, so as to be easily broken afterwards at these points. Then the tube is evacuated sufficiently down to a pressure of about 0.01 mm of mercury by proper heating. When thoroughly evacuated, the tube is sealed at its neck as shown in C, Fig. 1. Then it is weighed, and its volume is measured by immersing it in water contained in a measuring cylin-

1. These Memoirs, **15**, 1 (1932).

der. The volume of the sealed tube serves for the reduction of weighing in vacuo. The liquid is then introduced into the tube by breaking the neck of the tube at *b* in a certain liquid of known density in which the density is to be measured, as is shown by D in Fig. 1. Next the weight of the specimen in the liquid is measured. After the weight in the liquid of the glass tube containing the specimen, and the broken neck, both of which are perfectly filled with the liquid, has been measured in the usual way, the glass tube is broken along the filed line *a*, and the specimen is taken out of the tube. Then the weight in the liquid of all the parts of the broken glass tube, as shown by E in Fig. 1, is measured in the same way, and the weight of the specimen only in the liquid is obtained immediately. Finally after all the parts of the broken glass tube have been freed from the liquid, rinsed well in distilled water and well dried, their total weight in air is measured. From this total weight and that of the evacuated sealed glass tube containing the specimen, we can obtain immediately the net weight of the specimen. From this net weight of the specimen and its weight in the liquid of known density, the net density of the specimen is calculated. In the procedure above stated, the reduction of weighings in vacuo and the correction due to the change of the density of the liquid with temperature are, of course, to be made always.

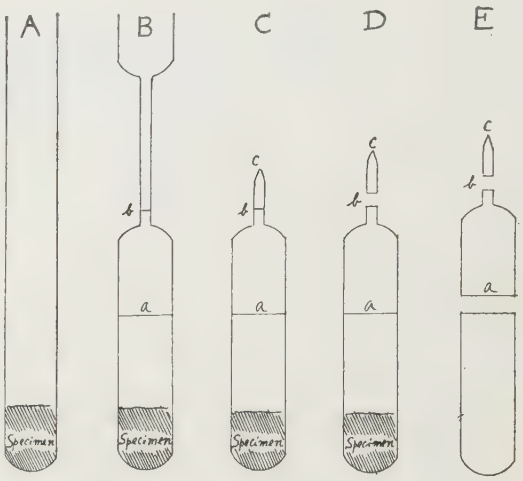


Fig. 1

When, for a given solid specimen, it is impossible to find a liquid which does not dissolve the specimen entirely, and we are obliged to employ a liquid of known density which does not act chemically with the specimen but dissolves a small part of it, then we are to find, anyhow, its apparent density by assuming that the dissolved quantity of the specimen is negligible.

Let m', S, C, D, E_1 and E_2 have respectively the following meanings:

m' : the weight of the specimen contained in the evacuated sealed glass tube,

S : the density of the liquid into which the specimen is to be dipped,

C : the weight of the evacuated sealed glass tube which contains the specimen,

D : the weight of the specimen and the broken pieces of the glass tube when they are immersed together in the liquid,

E_1 : the weight of all the broken pieces of the glass tube when they are immersed together in the liquid,

E_2 : the weight of all the broken pieces of the glass tube when they are well washed and dried.

Then the apparent density d of the specimen will be given by the following equation :

$$\frac{1}{d}=\frac{1}{m'S}\Big\{(C-E_2)-(D-E_1)\Big\}$$

$$=\frac{1}{S(C-E_2)}\Big\{(C-E_2)-(D-E_1)\Big\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

Next let m'', m_g, m_l, V and d_g have respectively the following meanings :

m'' : the mass of the specimen which still remained as the solid when it is immersed in the liquid,

m_g : the mass of the glass tube,

m_l : the mass of the liquid in the glass tube which dissolved a small part of the solid specimen,

V : sum of the volume of the solid specimen and the liquid contained in the glass tube,

d_g : the density of the glass.

Then

$$C=m'+m_g,\ D=(m''+m_g+m_l)-\left(V+\frac{m_g}{d_g}\right)S,$$

$$E_1=m_g-\frac{m_g}{d_g}S,\ E_2=m_g.$$

And the equation (1) becomes

$$\frac{1}{d}=\frac{1}{Sm'}\left[m'-\left\{(m''+m_g+m_l)-\left(V+\frac{m_g}{d_g}\right)S\right\}+m_g-\frac{m_g}{d_g}S\right]$$

$$=\frac{1}{Sm'}\Big\{(m'-m'')+(VS-m_l)\Big\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

If we represent the dissolved part of the solid specimen by Δm, then equation (2) becomes

$$\frac{1}{d}=\frac{1}{Sm'}\left\{\Delta m+(VS-m_l)\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(3).$$

When the real density of the specimen is D, and the volume of the liquid contained in the glass tube is V_s, then:

$$V=V_s+\frac{m''}{D}=V_s+\frac{m'}{D}-\frac{\Delta m}{D},$$

and the equation (3) becomes

$$\frac{1}{d}=\frac{1}{Sm'}\left\{\Delta m+V_sS+\frac{m'}{D}S-\frac{\Delta m}{D}S-m_l\right\}$$

$$=\frac{1}{D}+\left\{\frac{\Delta m}{m'}\left(\frac{1}{S}-\frac{1}{D}\right)+\frac{1}{Sm'}(V_sS-m_l)\right\} \quad \cdots\cdots\cdots(4).$$

When the solid specimen does not dissolve, $\Delta m=0$, and $V_sS=m_l$; and consequently d itself becomes to be the same as D from equation (4), as ought to be.

Now we must consider closely the mass m_l of the liquid part contained in the glass tube, in which the specimen is dissolved a little. Let S be the density of the pure liquid, and V_s be the volume of the solution when a small quantity m of the solid specimen is dissolved uniformly in the pure liquid of volume V', then the concentration of the solute in the solution will be m/V_s. We must enquire, before every thing, about the relation between the concentration and the corresponding density of the solution. Though it is a little different from the actual case usually, it is convenient to consider the ideal case that the volume of the solute and the solvent is additive in forming the solution. In this case the density of the solution D_s is calculated as follows:

$$D_s=\frac{SV'+m}{V'+\frac{m}{D}}=\frac{S\left(1+\frac{m}{SV'}\right)}{1+\frac{m}{DV'}}\fallingdotseq S\left\{1+\frac{m}{V'}\left(\frac{1}{S}-\frac{1}{D}\right)\right\}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(5).$$

When m is very small we can regard V' and D approximately equal respectively to V_s and d in the above equation. Then (5) becomes

$$D_s=S\left\{1+\frac{m}{V_s}\left(\frac{1}{S}-\frac{1}{d}\right)\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(6),$$

where m/V_s is the concentration of the solute in the solution. If we represent D_s in the form

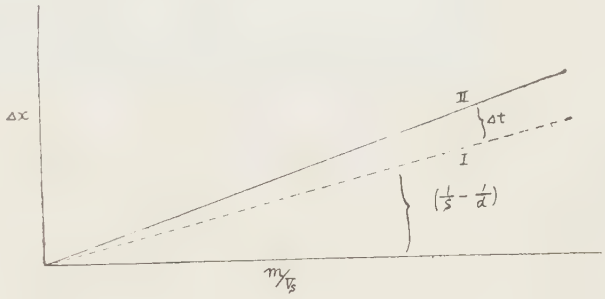
$$D_s=S(1+\Delta x),$$

then from equation (6) we obtain

$$\Delta x = \frac{m}{V_s}\left(\frac{1}{S} - \frac{1}{d}\right) \quad \text{.....(7),}$$

which shows that Δx is proportional to the concentration of the solute in the solution. This relation is represented by the dotted straight line I in Fig. 2. In actual case, this relation is somewhat different, but we may still assume with sufficient accuracy that Δx is proportional to m/V_s when the latter is small, as is shown by the straight line II in Fig. 2. Let the tangent value of the inclination of this straight line to the abscissa be t, and its excess to that of the straight line I be Δt, then we get

Fig. 2



$$1 + \Delta x = 1 + \left(\frac{1}{S} - \frac{1}{d}\right)\frac{m}{V_s} + \Delta t \frac{m}{V_s} \quad \text{.....(8).}$$

This relation is necessary in getting the correction to be added to the apparent density of the solid specimen, and a preliminary experiment is needed to get the value of Δt by measuring the densities of the solutions of various concentrations at a given temperature.

Next, in measuring the weight of the specimen and the broken pieces of the glass tube when they are immersed together in the liquid of density S, in the manner as shown by D in Fig. 1, the concentration of the dilute solution is different according to the position in the glass tube, and is generally smaller as we recede from the solid specimen. If we represent the coordinates of a point in the glass tube by (x, y, z), then the concentration c of the solution at that point will be represented as a certain function f of the coordinates of the point

$$c = f(x, y, z).$$

And the density of the solution at that point becomes from (8):

$$S + \left\{\left(\frac{1}{S} - \frac{1}{d}\right) + \Delta t\right\} S f(x, y, z) \quad \text{.....(9).}$$

Thus if we represent by dV_s the volume element in the solution, then the total mass of the solution m_1 which is contained in the glass tube is given by

$$m_i = \int\left[S + \left\{\left(\frac{1}{S} - \frac{1}{d}\right) + \Delta t\right\} S f(x, y, z)\right] dV_s$$
$$= S\int dV_s + \left\{S\left(\frac{1}{S} - \frac{1}{d}\right) + S\Delta t\right\}\int f(x, y, z) dV_s \quad \cdots\cdots\cdots (10),$$

where the integral is to be extended to the whole portion occupied by the solution in the glass tube. In the above equation the first integral is the volume occupied by the solution in the glass tube, and the second integral is nothing but the mass of the solute dissolved into the liquid contained in the glass tube, if we neglect the amount of the solute escaped through the narrow neck of the glass tube. Consequently equation (10) becomes

$$m_i = SV_s + \left\{S\left(\frac{1}{S} - \frac{1}{d}\right) + S\Delta t\right\}\Delta m \quad \cdots\cdots\cdots\cdots (11).$$

By using this value of m_i, we get

$$V_s S - m_i = -S\Delta m\left\{\left(\frac{1}{S} - \frac{1}{d}\right) + \Delta t\right\}.$$

And by inserting this relation into equation (4), we obtain

$$\frac{1}{d} = \frac{1}{D} + \left[\frac{\Delta m}{m'}\left(\frac{1}{S} - \frac{1}{D}\right) - \frac{\Delta m}{m'}\left\{\left(\frac{1}{S} - \frac{1}{d}\right) + \Delta t\right\}\right]$$
$$\fallingdotseq \frac{1}{D} - \frac{\Delta m \Delta t}{m'} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (12).$$

When the volume of the solute and the solvent is additive in forming the solution, Δt is nil as is stated, and the apparent density d becomes to be equal to the real density D, from the above equation, as is expected without any calculation.

When Δt is different from zero, as is actually the case, we get from (12)

$$D = d\left\{1 - d\Delta t\frac{\Delta m}{m'}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (13)$$

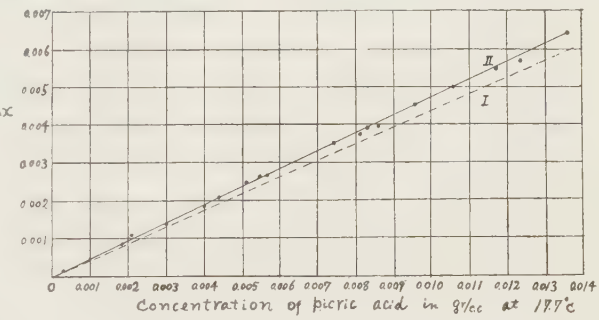
as the correction formula to obtain the real density D from the apparent density d.

Actually the apparent density d can be obtained by equation (1) as is stated before, and Δt by plotting the relation between the density and the concentration of the solute in the solution in the manner as is shown by Fig. 2. In order to get the dissolved amount Δm of the specimen, we collect in another proper vessel all the solution, excepting the solid part of the specimen, which is contained in the glass tube up to its broken neck, and measure its well mixed total volume V_s, and then its density by using a pyknometer. From the value of the density of the solution, we get the concentration of

the solute in the solution from the curve in Fig. 2, which is plotted in the preliminary experiment; and consequently from this value of the concentration and the volume V_s of the solution we get immediately the dissolved amount Δm of the specimen. Thus the correction to be made to the apparent density is readily calculated by equation (13) by giving the values of d, Δt and Δm thus obtained.

As an example the writers measured the net densities of crude and purified picric acid by the above method by immersing the specimen in distilled water which dissolves picric acid a little. Densities of the solutions corresponding to various concentrations of crude picric acid in aqueous solution were measured at 17.°7 C by means of a pyknometer, and the value of Δx was confirmed to be proportional to the concentration as is seen by the straight line drawn in full line in Fig. 3. The dotted straight line in the same figure represents the theoretical values calculated by assuming that the volumes of picric acid and the water is additive in forming the solution. The deviation of the observed values of Δx from those calculated is very small, and the value of Δt is found to be 0.0418 in this case. In the case of purified picric acid, the same experiment must be done, of course, with purified picric acid; but in addition to the smallness of the value of Δt, the dissolved amount of picric acid was very small both in crude and purified specimens, and was of the order of about 1/100 of the specimen employed. These facts show that the correction is very small in both cases. Moreover, as the value of Δt seems not to be much different in both cases, the writers employed the value of Δt found thus with crude picric acid too in the case of the purified one. As to the correction due to the variation of the density of water with temperature, we had only to take the value of the density of water at the temperature at which the measurement was performed as the value of S, the density of the liquid in which the specimen and the broken glass tube are immersed. As the sample of the picric

Fig. 3



acid was rather a porous one, it was ground into fine powder in a mortar before the measurement, so as to crush the enclosed vacant spaces as much as possible.

The results thus obtained are summarised in Table I and Table II. In every experiment the quantity of the picric acid employed was of the order of from 5 to 10 grams, and the probable error in the value of the density is estimated to be of the order of ± 0.001. For purified picric acid we get the mean value of 1.764^2 from six different measurements as its density; and this seems to be the most reliable value, though it is a little smaller than 1.767 given in Physikalisch-Chemische Tabellen of Landolt and Boernstein.

Table I. Crude picric acid

No. of experiment	Apparent density d	Real density D
1	1.7718	1.7704
2	1.7731	1.7717
3	1.7737	1.7725
4	1.7737	1.7714
5	1.7713	1.7703
Mean value	1.772^7	1.771^3

Table II. Purified picric acid

No. of experiment	Apparent density d	Real density D
1	1.7650	1.7632
2	1.7664	1.7651
3	1.7654	1.7642
4	1.7656	1.7645
5	1.7641	1.7630
6	1.7679	1.7652
Mean value	1.765^7	1.764^2

In conclusion the writers wish to express their hearty thanks to Professor Nozu for his kindness in preparing purified samples of picric acid for the present experiment.

The Effect of Sudden Heating on the Recrystallization of Metals

By Usaburo Yoshida, Saburo Nagata and Chikara Mitsuki

(Received April 20, 1936)

Abstract

In recrystallizing thin aluminium plates the effect of sudden heating to a recrystallization temperature belonging to the germinative temperature range upon the grain number was investigated, and it was found that the grain number is much greater in this case than when the plate is heated gradually up to the same recrystallization temperature. In the case of gradual heating, a few seeds or crystal nuclei which are to be grown to tolerable size afterwards in the germinative temperature range seem to be formed among an immense number of the recrystallized metal crystals already in the recrystallization temperature range lower than the germinative temperature.

Recently one of the writers and Koyanagi[1] published an opinion on the recrystallization of metals. In that occasion they assumed that the melting point of a more distorted crystal is lower than that of a less distorted one, and that the metal is easier to melt at its crystal boundaries than in the interior of individual crystals; and the process of recrystallization was considered to be the growth of almost undistorted perfect crystal nuclei formed in the melts at the crystal boundaries. This theory is of course a rough one, and must be subjected to closer examination from various standpoints concerning this phenomenon, which may request some refinement or modification at least. Following this purpose the writers began by examining the effect of sudden heating on the recrystallization of thin aluminium plate in the present experiment.

In order to ensure as far as possible a simultaneous sudden heating of the whole portion of the specimen, thin commercial aluminium plate of 1 m.m. thickness was used throughout the present experiment; and small test pieces having the areal size of 10 c.m. × 1.5 c.m. were cut out from the plate. The furnace used in the present experiment was a resistance type, consisting of a porcelain tube about one metre in length and five centimetres in diameter and a nichrome wire wound uniformly around it. The furnace was set vertically, and the lower end of the porcelain tube was plugged tightly with asbestos.

1. U. Yoshida and K. Koyanagi: These Memoirs, **18**, 9 (1935)

At the upper end of the tube a cover of asbestos plate is provided which facilitates the operation of opening and covering the upper end of the tube. In exposing a test piece to some high temperature, it was always hung free at the middle of the furnace by means of a wire hooked at the upper end of the porcelain tube; and the temperature to which the test pieces were exposed was measured by a thermocouple, whose one junction was placed at the middle part of the furnace. In the case of exposing a test piece to some required high temperature we must at first properly regulate the heating electric current through the nichrome wire of the furnace, so that the required high temperature is maintained steadily for a long time. After such a temperature is established in the furnace, we insert the test piece into the furnace as quickly as possible in the manner stated above by opening and covering the asbestos cover. This process can be done very quickly within the time of two or three seconds; and the resulting slight lowering of the temperature of the furnace caused by this process is recovered automatically to the initial steady temperature within several seconds.

In the present experiment all the test pieces of commercial aluminium plate were subjected to a preliminary annealing for 24 hours at the temperature of the furnace of 320°C; then they were elongated at room temperature, for various amounts of elongation ranging from 1 percent to 10 percent of the original length. Such cold-worked test pieces were next exposed suddenly, unless otherwise described, to various definite recrystallization temperatures one by one in the manner stated before; and the aspect of the test pieces thus recrystallized was revealed to the naked eye by etching the surface of the test pieces with some reagent after they were taken out of the furnace.

First of all the writers examined the relation between the recrystalization temperature, the amount of the cold-working of the test piece before recrystallization, and the number of the crystals formed by recrystallization, which were contained in the unit areas of the surface of the test pieces. The results obtained are tabulated in Table I. For the cold-working the writers elongated the specimens by stretching, and the percentage of elongation of the original length was taken as the amount of cold working, as is given in the table. Here it must be noted that, though the time of recrystallization which is needed to complete the grain growth is different according to the

amount of the cold working and the recrystallization temperature, it was taken for a time long enough that the grain growth was ended and the grain boundaries became entirely sharp. The counting of the crystal number was made at six different portions of each specimen, and the average number per square centimetre is given in the table.

Table I

Number of grains/cm², (sudden heating)

Elongation in % / Recrystallization temperature	10	7	6	3	2
645°C	145	64	43	31	31
630°	49	54	39	28	27
600°	114	47	40	18	17
550°	27	38	11	6	—
500°	90	15	3	—	—

As is seen in Table I, the grain number per unit area increases, for a given recrystallization temperature, with increased amount of cold working, as is usually the case with gradual heating ; and it increases, for a given amount of cold working, with increased recrystallization temperature. This temperature relation of the grain number is entirely different from the case of gradual heating, in which case the grain number per unit area does not increase, but rather tends to decrease with increased recrystallization temperature due to the growth of the crystals. The grain number per unit area is much greater in the case of sudden heating than when the temperature of the furnace is raised gradually to the maximum recrystallization temperature. To make clear this point, another recrystallization experiment with the test pieces having the same size as above was carried out by raising the temperature of the furnace gradually from the room temperature to the maximum recrystallization temperature of 640°C. In this case the maximum temperature was attained in about 4 hours, and the test pieces, which were inserted into the furnace at the beginning, were still maintained at this maximum recrystallization temperature for 20 hours. The values of the grain numbers per unit area of the surface of the test pieces, averaged with a number of test pieces with equal amount of cold working respectively, are tabulated in Table II.

With the recrystallization temperature as high as at least 600°C or more, the grain growth is almost completed with prolonged heating, and the average grain number is almost independent from the maximum recrystallization temperature. The values tabulated in table II

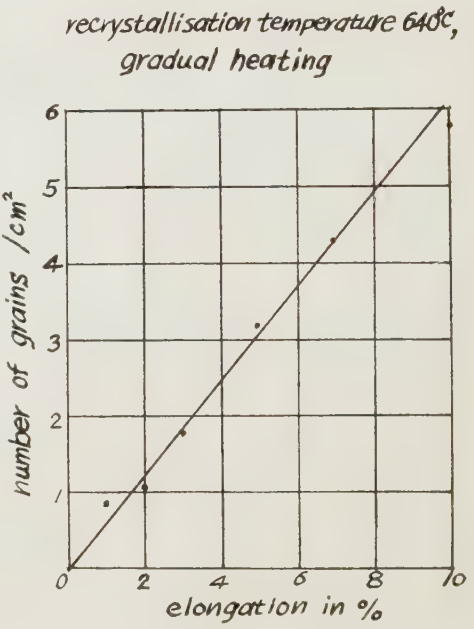
Table II

Maximum recrystallization temperature 640°C, (gradual heating)

Elongation in %	10	7	5	3	2	1
Number of grains /cm²	5.8	4.3	3.2	1.8	1.1	0.9

are shown graphically in Fig. 1, and a linear relation between the elongation and the number of grains per unit area is revealed clearly. The grain number observed in the present case is very much smaller and is of different order compared with the case of sudden heating. This point is very important, and we must be very cautious on this point in experimenting with the effect of temperature on the recrystallization phenomena; otherwise the results are liable to become entirely erroneous. For example if the thickness of the test piece is too large, it will be almost impossible to carry out the experiment with the effect of sudden heating, owing to the difficulty in applying a definite recrystallization temperature very quickly to the whole portion of the specimen.

Fig. 1



In the case of sudden heating to a recrystallization temperature, the number of grains per unit area increased with the recrystallization temperature for a given amount of cold working as stated before. But this is different from the recrystallization schema given by Czochralski[1] and

1. J. Czochralski: Moderne Metallkunde, 130, (1924).

other authors, where the grain size (not the grain number) is considered to become larger with increase of the recrystallization temperature. This discrepancy seems to be caused by the fact that the conditions under which they made their experiments rather belong to those of gradual heating, but not to those of sudden heating. In the present experiment the recrystallization temperature was higher than 500°C, and the crystal grains of aluminium obtained by recrystallization were rather large, so that the grain number could be counted by the naked eye. Consequently the recrystallization temperatures higher than 500°C for aluminium seems to be generally in germinative temperature range[1] in which exaggerated grain growth occurs, for the amount of cold working stated before, though the germinative temperature depends upon the amount of cold working. Thus it may be said, in the case of sudden heating to a recrystallization temperature, that the grain size diminishes with the increase of the recrystallization temperature, so far as the recrystallization temperature remain in the germinative temperature range.

Next the writers made a rough estimate of the time necessary for germination after the test piece had been subjected suddenly to a recrystallization temperature. For this purpose several test pieces were prepared for a given amount of elongation and a given recrystallization temperature. These test pieces were subjected separately to the same recrystallization temperature for different durations of time, and the state of the progress of germination was made perceptible to the naked eye by etching the surface of the test pieces with some reagent after they had been taken out of the furnace. The writers classified the recrystallized test pieces into three groups: 1) "no indication of germination," 2) "indication of germination" and 3) "complete germination." Among these three, 1) and 2) can be distinguished with adequate sharpness in recrystallization time, and the time necessary for the state 2) changes most regularly with the amount of cold working and the recrystallization temperature. On the contrary the time necessary for the state 3) changes very irregularly with tolerable fluctuation, which makes it almost impossible to set a sharp distinction between the states 2) and 3) with recrystallization time. Moreover X-ray examination indicates that already in the state 2) most of the metal crystals have grown into fairly large ones, being disclosed by the appearance of only a few Laue-spots of strong

1. Jefferies and Archer: The science of metals, 108 (1924).

intensity. Thus the time necessary to bring about the state of "indication of germination" was looked for roughly as the measure of the time which was necessary for germination to be almost completed, and it will be called "the germination time" hereafter. In the present experiment the germination time defined thus was measured in the unit of one minute; and the reciprocal of the germination time, which

Fig. 2

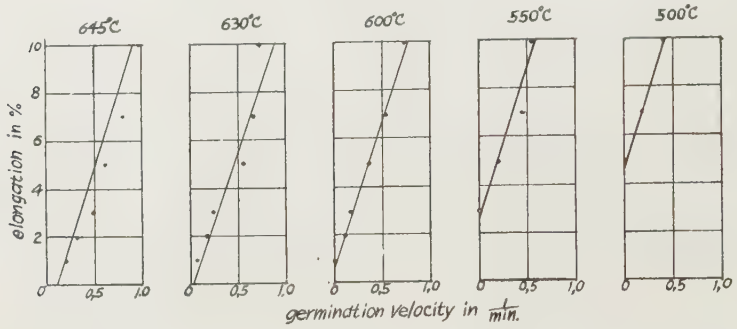
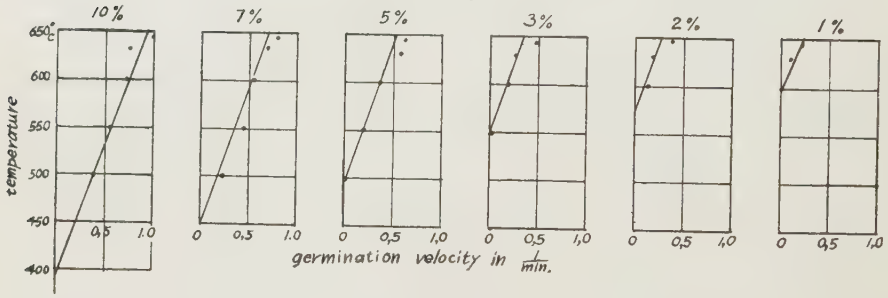


Fig. 3



may be called "the germination velocity," was considered in relation to the amount of cold working and to the germinative temperature. The relation between the elongation of the specimen and the germination velocity for various germinative temperatures is shown in Fig. 2, and the relation between the germinative temperature and the germination velocity for various amounts of cold working is represented in Fig. 3. From these figures it will be easily seen that the germination velocity increases almost linearly both with the amount of cold working and the germinative temperature. If we represent the germination velocity by v, the elongation by S, the melting point of aluminium by T_m (657°C), and the germinative temperature by T, then the above linear relation is found to be expressed very simply by the following empirical formula:

$$v=0.082\{S-0.042(T_m-T)+1.84\} \quad \ldots\ldots(1).$$

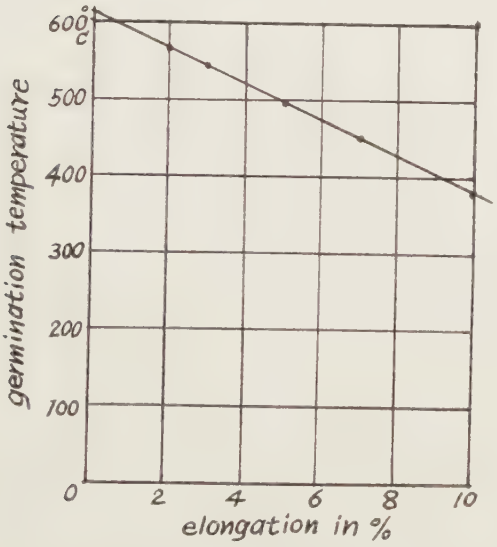
The relation given by this formula is represented by the straight lines drawn in Figs. 2 and 3, which fit fairly well with the observation.

It will be seen from the observation on germination velocity that the germination takes place at a temperature higher than a certain temperature, which may be called "the germination temperature," and that the germination temperature decreases with increase of the amount of cold working as in the case of the annealing temperature. If we represent by T_g the germination temperature, then the relation between T_g and the amount of cold working will be readily obtained by putting $v=0$, and $T=T_g$ in the above equation as follows,

$$T_m-T_g=\frac{S+1.84}{0.042} \quad \ldots\ldots(2).$$

This relation is shown by the straight line in Fig. 4. Here it must be noted that the germination temperature, like the annealing temperature, is probably not a very definite one, and that more prolonged heating than in the present experiment may cause further development of germination. And consequently the straight line drawn in Fig. 4 ought to be understood to represent the higher limit of the rather indefinite germination temperature.

Fig. 4



From the standpoint of the theory stated before, the writers next attempted to formulate roughly the relation between the grain number, amount of cold working, and the germinative temperature to which the specimen was subjected for its germination. In that theory it was assumed that the crystal boundaries were easier to melt than in the interior of the crystal, and that the melting point of a crystal is lower with increased distortion, and that the recrystallization is nothing but the growth of the perfect crystal nuclei formed in the melt. It seems very probable that the grain growth to considerable size by recrystallisation does not take place

directly by the consumption of the initial minute crystallites which have suffered some distortion by the previous cold working, but that it is performed by repeating several times (two times probably) the process of melting and crystallization, as is manifested by rather even or uniform grain growth of the metal crystallites at a rather lower recrystallization temperature. This seems to the writers to be due to the fact that at low temperature in the case of gradual heating or at early stage of recrystallization in the case of sudden heating, the crystals formed by recrystallization are not entirely free from strain; and thus they have still the predisposition to be recrystallized on further heating. When the temperature in the case of gradual heating becomes higher or when the stage of recrystallization advances further in the case of sudden heating, the specimen will get sufficiently soft, so that the new crystals formed then become almost entirely free from strain; and consequently they grow further and further on continued heating to exaggerated grain sizes by consuming the older crystals having more strains than themselves. The stage of germination seems to be understood to occur in such a manner.

Though the phenomena of recrystallization of metals are very complicated as stated above, it seems almost certain that the presence of distortion in the crystal lattice is its primary cause. When a small part of the crystal lattice is distorted the potential energy of the atoms in that part will become higher than the normal value in the same part, which is peculiar to the perfect crystal lattice. Next let us consider a small volume element in a distorted crystal lattice and the excess of the potential energy per unit volume at that part, in reference to the perfect undistorted lattice is ϕ. Then it seems not to be entirely absurd to assume that the number of localities $Nd\phi$ per unit area of the specimen which has a value of the excess potential energy ranging between ϕ and $\phi+d\phi$ is governed by a probability law, and that the localities having a larger value of ϕ increase in number with increase of the amount S of cold working.

This consideration may be roughly expressed by

$$Nd\phi = Ce^{-\frac{\phi}{\alpha S}}d\phi \quad \text{.....(3).}$$

Next assume that the lowering of the melting point at a locality in the crystal is proportional to the amount of the excess potential energy ϕ there. Thus if we represent by T the melting point at that locality, and by T_m the melting point of the perfect crystal, then equation (3) becomes

$$N \cdot dT = Ce^{-\frac{T_m - T}{\alpha S}} dT \quad \text{.....(4)},$$

where C and α are constant.

Now let us consider the case when the specimen is suddenly subjected to a recrystallization temperature, then the number of the crystal nuclei N' per unit area which is supposed to be formed at melted localities will be given by

$$N' = \int_0^T N \cdot dT = \int_0^T Ce^{-\frac{T_m - T}{\alpha S}} dT = \alpha S C \left(e^{-\frac{T_m - T}{\alpha S}} - e^{-\frac{T_m}{\alpha S}} \right).$$

If we consider the case when the recrystallization temperature is not very far from the normal melting point of the metal as is actually the case, then the second term in the bracket may be neglected in comparison with the first term, and the above equation becomes

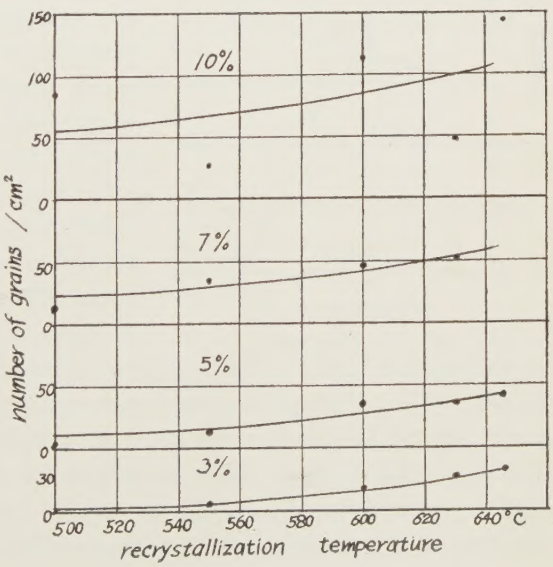
$$N' = \alpha S C e^{-\frac{T_m - T}{\alpha S}}.$$

Generally let us assume that C is not independent of the amount of the cold working, and represent αSC by a new constant K which is a function of S, then we obtain

$$N' = Ke^{-\frac{T_m - T}{\alpha S}} \quad \text{.....(5)},$$

which represents the relation between the number of the nuclei per unit area of the surface of the specimen, the recrystallization temperature to which the specimen is subjected suddenly, and the amount of the cold working which is represented in the present case by the elongation expressed in % of the original length. As when the crystals are fully grown the number of the crystal nuclei becomes entirely equal to the grain number, the number of the crystal grains per unit area of the surface of the specimen is given by the above equation too.

Fig. 5



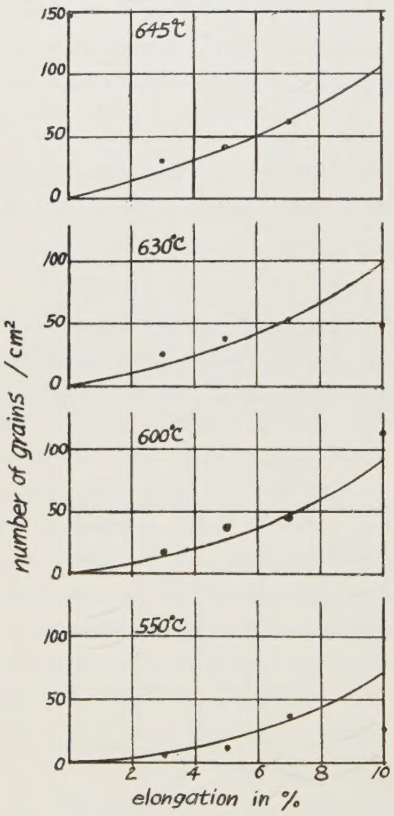
The constants α and K in the above equa-

tion were so chosen as to fit as near as possible to the grain number observed in the present experiment and tabulated in Table I for the case of sudden heating, and for their best values the writers obtained the following values by trial:

$$a=21.0, \quad K=21.0\times e^{0.18S}.$$

The values of N' calculated by putting these values to equation (5) are plotted in curves in Figs. 5 and 6. Though the agreement between the calculated and the observed values represented by dots is not satisfactory, the general tendency seems to be well expressed by the consideration stated before.

Fig. 6



It is already stated that the number of the crystal grains formed by recrystallization is very much different according as the specimen is subjected suddenly to a definite recrystallization temperature or the recrystallization temperature is raised gradually from the room temperature to a final value, even when this final value is the same as the recrystallization temperature in the case of sudden heating. This fact indicates that the grain number depends more upon the heating at lower temperatures than the final maximum temperature in the case of gradual heating. To make clear this point the writers carried out another experiment. Test pieces of commercial aluminium plate having the same size as stated before were preliminary annealed for 24 hours at the temperature of 320°C, then they were elongated 7% of the original length. After these test pieces were subjected suddenly to the first lower recrystallization temperature separately for about 5 hours, also their temperature was raised gradually to the second higher recrystallization temperature of 645°C for about 20 hours separately. The number of the grains thus formed per square centimetre of the surface

of the specimen, corresponding to various first recrystallization temperatures are tabulated in Table III.

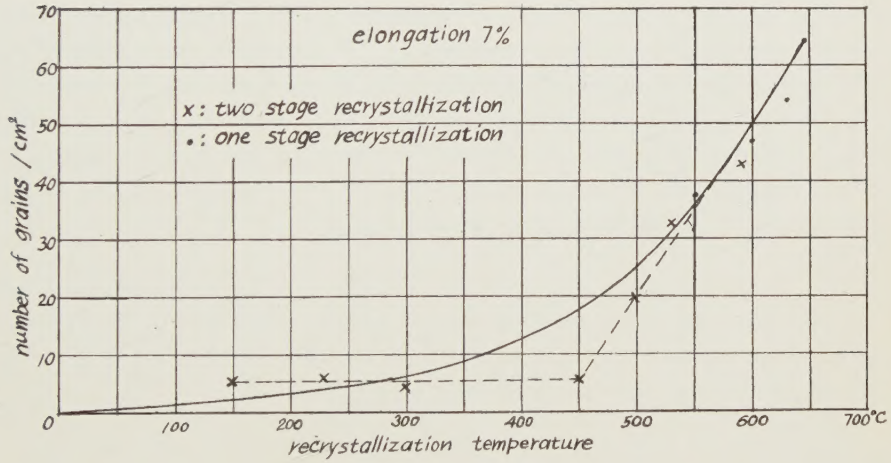
Table III

First recrystallization temperature, °C	590°	530°	500°	450°	300°	230°	150°
Number of grains/cm²	43	33	20	6.0	4.6	5.8	5.5

Elongation 7%, second recrystallization temperature 645°C.

In order to compare the results obtained now with two stage recrystallization to those obtained before with one stage recrystallization, the results obtained in both cases are represented together in Fig. 7, by regarding the first lower recrystallization temperature in the case of two stage recrystallization as the same as the recrystallization temperature in the case of one stage recrystallization. Nice amalgamation of the two results, as is seen in the figure, indicates clearly that the number of the crystal grains in the case of two stage heating, when

Fig. 7



they are pretty well developed after being exposed to the second higher recrystallization temperature belonging to the germinative temperature rang, is primarily determined by the first lower recrystallization temperature. This seems to indicate that the healthy perfect crystal nuclei which have the ability to grow further and further in the germinative temperature range, or the localities where such nuclei are to be formed are already seeded in the first stage of lower re-

crystallization temperature; and that the germination is nothing but the growth of such healthy crystal nuclei. In Fig. 7 the curve drawn in full line is that calculated by equation (5), and the broken straight lines are so drawn that they may fit best to the observation points. These two broken straight lines intersect at about 450°C, which is nothing but the germination temperature for the elongation of 7% as will be seen from Fig. 4. Thus it may be said that in the germinative temperature range, seeding and germination of the healthy crystal nuclei occur simultaneously in the case of sudden heating, and that in the recrystallization temperature range lower than the germination temperature seeding only without germination takes place. Though the number of the crystals recrystallized in such a lower recrystallization temperature range is immense, the number of the seeds formed in such a stage is very small only of the order of about 5 per square centimetre on the surface of the aluminium plate in the present case. This number coincides roughly with that obtained by extrapolating the theoretical curve in Fig. 7, and also with that obtained in the case of gradual heating for 7% elongation, which was about 4 as is given in Table II. This seems to indicate that in the case of gradual heating up to the germinative temperature range, only a few seeds formed among an immense number of the crystals recrystallized in the early stage of lower temperature grow to very large crystals in the later germinative temperature range. The question of why a much larger number of grains are formed in the case of sudden heating in the germinative temperature range than in the case of gradual heating seems to be understood in such a manner.

In conclusion, the writers' sincere thanks are due to the Hattori Hoko Kai for their grant given to one of the writers (U. Y.), with the aid of which this research was accomplished.